International Journal of Emerging Technology and Advanced Engineering
Website: www.ijetae.com (ISSN 2250-2459, Volume 2, Issue 8, August 2012)

Study of Optical Properties of Tokamak Plasma

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***Abstract*— In a tokamak plasma, the electrons are confined by total helicoidal magnetic field and they are subject to Lorentz's force which allows them to gyrate around the field lines. These gyrating electrons emit electromagnetic radiations known as electron cyclotron emission ECE at the cyclotron resonance frequency and its harmonics. These frequencies fall in the millimeter wave region of electromagnetic spectrum. Radiometry of electron cyclotron emission (ECE) can be used to determine the electron temperature of the plasma. In such middle, the optical depth depends on the electron density and on the electron temperature, decreases with increasing harmonic number.**

***Keywords*—Electron, cyclotron, emission, harmonics, radiometry, optical depth, harmonic number.**

I. INTRODUCTION

The injection of electron-cyclotron (EC) waves is nowadays a well-established method for coupling energy to plasma electrons in modern fusion devices, with primary applications the plasma heating (ECRH) and the generation of non-inductive current drive (ECCD). These applications based on the interaction of electromagnetic (EM) wave with resonant electrons motion when they gyrate around magnetic field lines. At the same time, Because of this accelerated motion electrons emit electromagnetic radiation at the cyclotron resonance angular frequency and its harmonics.

Electron cyclotron emission (ECE) measurements are used in most tokamaks to obtain highly resolved measurements of the electron temperature. A good time resolution and a high spatial resolution, with a full plasma coverage provided at the same time in case of multichannel receivers.

II. ELECTRON CYCLOTRON WAVE IN MAGNETIZED PLASMA

To describe the propagation of electron cyclotron waves in plasma is generally used the cold plasma approximation [2]. In this approximation the plasma pressure is assumed very small compared to the magnetic pressure $\beta \ll 1$. In this case the thermal motion of electrons may be negligible in terms of oscillations of the wave $v_\varphi \gg v_{th}$ where v_φ is the phase velocity of the wave and v_{th} is an electron thermal velocity and the Larmor radius is small compared to the wave length [3]. Considering plane wave solutions of Maxwell's equations, such as fluctuating quantities vary as $exp\,(i\,(\vec{\boldsymbol{k}}\,.\vec{\boldsymbol{r}} - \omega t))$. In Fourier space, we can find a wave equation of the form [4]:

$$k^2\vec{\boldsymbol{E}} - \vec{\boldsymbol{k}}(\vec{\boldsymbol{k}}\vec{\boldsymbol{E}}) - \left(\frac{\omega^2}{c^2}\right)\vec{\boldsymbol{D}} = 0 \qquad (1)$$

Where $\vec{\boldsymbol{k}}$ is the wave vector, $\vec{\boldsymbol{D}} = \bar{\bar{K}}\vec{\boldsymbol{E}}$ is the electrical induction vector, $\bar{\bar{\boldsymbol{K}}}$ is the dielectric tensor [2] [5], [6], $\vec{\boldsymbol{E}}$ is the vector of wave electric field. If the refractive index is written $\vec{\boldsymbol{N}} = \frac{\omega}{c}\vec{\boldsymbol{k}}$, the equation (1) can conduct to resolving the dispersion equation which may take the form :

$$AN^4 + BN^2 + C = 0 \qquad (2)$$

With $A = Ssin^2\theta + Pcos^2\theta$, $B = RLsin^2\theta + PS(1 + cos^2\theta)$ and $C = PRL$. In the case of perpendicular propagation to magnetic field ($N_{II} = 0$). We obtain two solutions of equation (2) for the perpendicular refractive index, which can be written:

$$N_O^2 = P = 1 - \frac{\omega_p^2}{\omega^2}, \qquad (3)$$

$$N_X^2 = \frac{S^2 - D^2}{S} = 1 - \frac{\omega_{pe}^2}{\omega^2}\frac{(\omega^2 - \omega_{pe}^2)}{(\omega^2 - \omega_{pe}^2 - \omega_{ce}^2)} \qquad (4)$$

IJETAE
EXPLORING RESEARCH AND INNOVATIONS

International Journal of Emerging Technology and Advanced Engineering
Website: www.ijetae.com (ISSN 2250-2459, Volume 2, Issue 8, August 2012)

These transverse electromagnetic solutions are well known by the names of ordinary mode (O-mode) and extraordinary mode (X mode) [7]. The first mode does not have any resonance and propagate for $\omega > \omega_{pe}$ because of the cut-off and the second one has two cut-offs and two resonances. According to the phase velocity ω/k, it decomposes in fast (F) and slow (S) as shown in Fig 3. The two branches of propagation (ordinary and extraordinary) appear and we can see that the ordinary mode propagates for frequencies such that $\omega > \omega_{pe}$. The extraordinary mode is propagated for $\omega_L < \omega < \omega_{uh}$, evanescent for $\omega_{uh} < \omega < \omega_R$. It becomes propagative when $\omega > \omega_R$. With ω_R, ω_L are the cutoff frequencies of the X mode, called right and left modes, defined by:

$$\omega_{R,L} = \frac{1}{2}\left[\mp\omega_c + \left(\omega_c^2 + 4\omega_p^2\right)^{1/2}\right] \qquad (5)$$

The X mode has a cold resonance ($N_\perp \to \infty$), given by:

$$\omega_{uh} = \sqrt{\omega_c^2+\omega_p^2} \qquad (6)$$

This resonance is called upper hybrid (UH) is not available if $\omega > \omega_c$. There is also a lower hybrid resonance [8], it is well below the electron cyclotron frequency domain and therefore not interfere here.

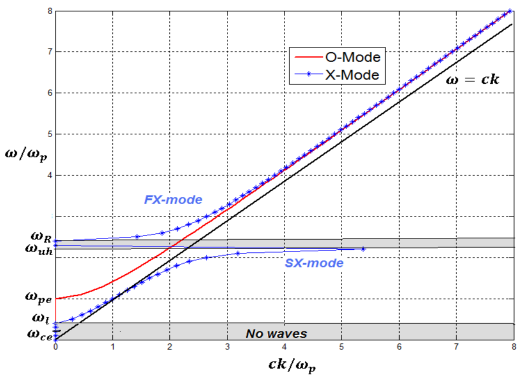

Figure 1: Dispersion diagram of O-mode and X-mode

III. ABSORPTION OF EC WAVES

We take the viewpoint of geometrical optics by considering a plane monochromatic wave type $\vec{E}(\vec{r},t) = \vec{E}(\vec{k},\omega)\, exp\{i[\vec{k}.\vec{r} - \omega t]\}$ for which one trying to describe the dissipation by introducing the concept of absorption coefficient. For there to be absorption, it is necessary that $k = k' + ik_a''$ avec the imaginary part of wave vector $k_a'' = (\omega/c)N'' \neq 0$. Then the absorption coefficient [7] is given by

$$\alpha = -2k_a''.\frac{\vec{v_g}}{v_g} \qquad (7)$$

With $\vec{v_g} = \frac{d\vec{r}}{dt}$ is the group velocity.

For the explicit calculation of the absorption coefficient, we introduce another approach based on energy conservation, using the anti-Hermitian part of the dielectric tensor. Poynting's theorem [9] writes:

$$\frac{\partial W_{0,t}}{\partial t} + \vec{\nabla}.\vec{S}_{0,t} = -\vec{j}_t.\vec{E}_t \qquad (8)$$

Where $\partial W_{0,t}/\partial t$, the instantaneous energy density contains the magnetic $|\vec{\mathbf{B}}_t|^2/(2\mu_0)$ and electrostatic $\frac{1}{2}\varepsilon_0|\vec{\boldsymbol{E}}_t|^2$ energies respectively. $\vec{\boldsymbol{S}}_{0,t}$ is the instantaneous Poynting vector in vacuum describing the flow of electromagnetic energy. The source term $-\vec{\mathbf{j}}_t.\vec{\mathbf{E}}_t$, describes the interactions of the wave with the plasma. By performing the time average over a few periods of oscillations $\langle\vec{\mathbf{E}}_t\rangle t = E_1(\vec{\boldsymbol{r}})\, exp(i\vec{\boldsymbol{k}}.\vec{\boldsymbol{r}})$, and separating explicitly the hermitian and anti-hermitian parts of dielectric tensor introduced into the source term, we can be extracted from equation (8) the absorption coefficient as:

$$\alpha = \frac{\varepsilon_0\omega\vec{E_1^*}\bar{\bar{K}}_a\vec{E_1}}{|\vec{S}|} \qquad (9)$$

Where $\vec{\boldsymbol{E}}_1^*$ is the complex conjugate of $\vec{\boldsymbol{E}}_1$ and $\vec{\boldsymbol{S}} = \vec{\boldsymbol{S}}_0 + \vec{\boldsymbol{Q}}_s$ with $\vec{\boldsymbol{S}}_0 = \frac{1}{4\mu_0}Re\left(\vec{\boldsymbol{E}}_1^* \wedge \vec{\boldsymbol{B}}_1 + \vec{\boldsymbol{E}}_1 \wedge \vec{\boldsymbol{B}}_1^*\right)$ and $\vec{\boldsymbol{Q}}_s = -\frac{1}{4}\varepsilon_0\omega\vec{\boldsymbol{E}}_1^*\frac{\partial\bar{\bar{K}}_h}{\partial k}.\vec{\boldsymbol{E}}_1$.

A. Optical Deph (Optical thickness):

A useful quantity is the optical depth τ [1], [8], [10], which is defined as the integral of the absorption coefficient α along the trajectory s of the wave: $\tau = \int -\alpha \, ds$. The total absorbed power P_{abs} in the plasma can then be written as

$$P_{abs} = P_{inj}(1 - exp(-\tau)) \qquad (10)$$

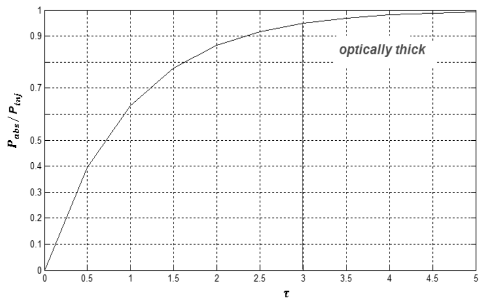

Figure 2: The fraction of absorbed power as a function of optical depth [9].

We can see an illustration of the function P_{abs}/P_{inj} on the Figure 2 where we define that the plasma is optically thick when τ > 3, i.e. when the fraction of absorbed power $P_{abs}/P_{inj} > 95\%$.

The relation of resonance is given by the relativistic cyclotron resonance condition of energy exchange between the wave electron cyclotron and plasma as follows:

$$\gamma - k_{II} v_{II} - n\frac{\omega_{ce}}{\omega} = 0 \qquad (11)$$

The term $k_{II} v_{II}$ describes longitudinal Doppler shift [3]. The term $n\omega_{ce}/\omega$ describes the gyration of the electron; n is the order of the harmonic excited.

TABLE I
The optical depth of EC waves [8]

	Mode	**Expression**
$n \geq 1$	O-mode-⊥	$\tau = \frac{\pi^2 n^{2(n-1)}}{2^{n-1}(n-1)!} N_O^{2n-1} \left(\frac{\omega_p}{\omega_c}\right)^2 \left(\frac{v_t}{c}\right)^{2n} \frac{R}{\lambda}$
$n \geq 2$	X-mode-⊥	$\tau = \frac{\pi^2 n^{2(n-1)}}{2^{n-1}(n-1)!} A_n \left(\frac{\omega_p}{\omega_c}\right)^2 \left(\frac{v_t}{c}\right)^{2(n-1)} \frac{R}{\lambda}$
		With $A_n = N_X^{2n-3}\left(1 + \frac{\left(\frac{\omega_p}{\omega_c}\right)^2}{n(n^2-1-\omega_p^2/\omega_c^2)}\right)$
$n = 1$	X-mode oblique	$\tau = \pi^2 N_X^5 \left(1 + \frac{\omega_p}{\omega_c}\right)^2 \left(\frac{\omega_c}{\omega_p}\right)^2 \left(\frac{v_t}{c}\right)^2 cos^2\theta \frac{R}{\lambda}$

The table 1 present the optical depths of a plasma slab in which the magnetic field varies as $B \sim 1/R$ and we have:

1- For the O-mode, the optical depth is given for perpendicular propagation and for all harmonics n = 1, 2.

2- Similarly for the X-mode and the harmonics $n \geq 2$.

3- The optical depth for the fundamental harmonic $n = 1$ of the X-mode is given for oblique propagation.

In the table, $v_{th} = (k_B T_e/m_e)^{1/2}$ is the thermal velocity of the electrons. In most current ECRH tokamak experiments either the fundamental O-mode or second harmonic X-mode are employed resulting in complete single pass absorption. As is shown on Figure 3, the optical depth for the second harmonic X-mode is the largest one.

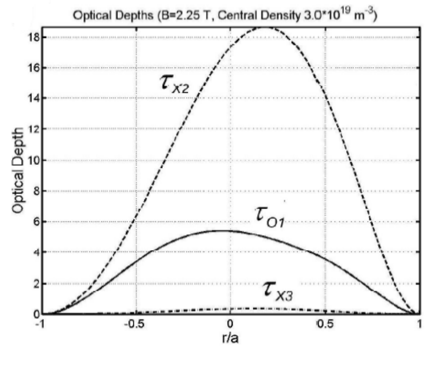

Figure 3: The optical depth $\boldsymbol{\tau}$ for O1, X2, X3 modes [10].

IV. ELECTRON CYCLOTRON EMISSION

In a tokamak, plasma is confined and stabilized by combination of an internal poloïdal magnetic field and a strong external applied toroidal magnetic field. The electrons , subject to Lorentz's force gyrate around the field lines at frequency ($\gamma = 1$, a non relativistic case):

$$f_c = eB/2\pi m_e \tag{12}$$

The gyrating electrons emit electromagnetic radiations known as electron cyclotron emission ECE at its harmonics. These frequencies fall in the millimeter wave region of electromagnetic spectrum. This radiation from electrons will be responsible of radiated energy loss from fusion plasmas. In a tokamak, the toroïdal magnetic field is generated by external coils $B_t = B_t(R) \propto 1/R$, where R is the major radius. This spatially varying magnetic field causes the electron to emit cyclotron radiation at frequencies corresponding to their radial location in plasma. A region of plasma that emitting ECE, at its cyclotron frequency is also an absorber of radiation at the same frequency incident on its intensity of the radiation

At low frequencies, the plasma is optically thick. Any radiation is reabsorbed. The plasma is like a black-body [12]. The black-body intensityI_{BB}, radiated power per unit area and unit solid angle, is given by Plank's law.

$$I_{BB} d\omega = \frac{h\omega^3}{8\pi^3 c^3} \frac{1}{e^{\frac{h\omega}{kT_e}} - 1} d\omega \tag{13}$$

Where $d\omega$ is the observable angular frequency interval. For tokamak plasma $hv < 1meV,\ kT > 10\ eV$), $hv \ll kT$ and hence the Rayleigh-Jeans approximation holds:

$$I_{BB}(\omega) = \frac{\omega^2 kT_e}{8\pi^3 c^2} \tag{14}$$

In these cases the plasma is said to be optically thick for that radiation. So *Te* can be estimated from the intensity.

As is shown on the Figure 4, the most important intensity corresponds to second harmonic for either O-mode or X-mode. In other cases where the absorption is not strong, the plasma is said to be optically thin and the ECE intensity from that layer is a function of its electron density as well as temperature.

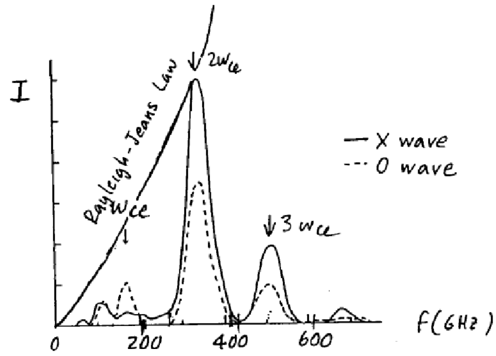

Figure 4: The intensities emitted for harmonics n=1, 2, 3of O-mode and X-mode [11].

At high frequencies, reflections from the walls become important and the intensity written as:

$$I(\omega) = I_{BB}(\omega) \frac{1 - e^{-\tau(\omega)}}{1 - re^{-\tau(\omega)}} \tag{15}$$

Where τ is the optical thickness and *r* is the reflection coefficient.

A. The Spectrum of Electron Cyclotron Emission

For perpendicular direction to the magnetic field, X waves dominate and the spectrum of electron cyclotron emission is obtained using Fourier transform spectroscopy [11].

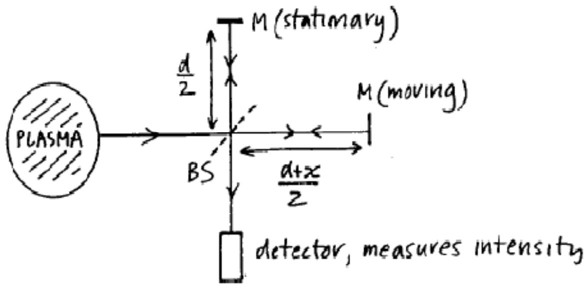

Figure5: Scheme for measuring the intensity of wave emission from plasma fusion (Michelson interferometer systems).

Suppose plasma emits a single frequency and the field at detector due to beam reflected off stationary mirror

$$E_1 = A\cos(kd - \omega t) \tag{16}$$

Field at detector due to beam reflected off moving mirror

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$$E_2 = A cos(k(d + x) - \omega t) \quad (17)$$

The output of the square-law detector is

$$(E_1 + E_2)^2 = 4A^2 cos^2\left(k\frac{x}{2}\right) cos^2\left(k\left(d + \frac{x}{2}\right) - \omega t\right)$$

$$= 4A^2 cos^2\left(k\frac{x}{2}\right)\frac{1}{2}\left(1 + cos2\left(k\left(d + \frac{x}{2}\right) - \omega t\right)\right) \quad (18)$$

The low-frequency part is

$$V = 2A^2 cos^2\left(k\frac{x}{2}\right) = A^2(1 + cos\, kx) \quad (19)$$

Express this as an intensity we find $I = S(k)(1 + cos\, kx)$. The plasma emits a range of frequencies. Using a similar approach we write:

$$I(x) = \int_0^\infty S(k)(1 + cos\, kx)dk \quad (20)$$

Where $S(k)$ is the spectrum. In a measurement, $I(x)$ is recorded for a range of x, the average value is subtracted out leaving what is called the interferogram. This is the Fourier transform

$$Int(x) = \int_0^\infty S(k). cos\, kx\, dk \quad (21)$$

To obtain the spectrum $S(k)$; we carry out the inverse transform

$$S(k) = \frac{2}{\pi}\int_0^\infty Int(x). cos\, kx\, dx \quad (22)$$

V. SUMMARY

The electrons, in a tokamak plasma are confined by total helicoidal magnetic field gyrate around the field lines. These gyrating electrons emit electromagnetic radiations known as electron cyclotron emission ECE at the cyclotron resonance frequency and its harmonics. These frequencies fall in the millimeter wave region of electromagnetic spectrum.

The main diagnostic aim of time-resolved spectroscopy of electron cyclotron emission is to determine the evolution of the electron temperature profile as a function of time. The toroidal magnetic field and the aspect ratio of fusion devices such as tokamaks determine the spectral range of the EC emission.

The optical depth decreases with increasing harmonic number and depends on the electron density and, in a hot plasma, on the electron temperature. In most fusion plasmas, the first harmonic in ordinary mode and the first and second harmonics in extraordinary mode are optically thick in the bulk plasma.

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