International Journal of Computer and Electrical Engineering, Vol. 2, No. 5, October, 2010
1793-8163

Study of the Bifurcation Generated by a Phase Transition. Application to a Nonlinear Capacitor

T. Benouaz, F. Belkhouche and N. Ghouali

***Abstract*—The bifurcation characterized by a phase transition, ie a temperature where the material structure changes qualitatively exists in a number of dielectrics. The Landau model for interpreting this kind of bifurcation, the degree of nonlinearity is controlled by temperature and allows the realization of nonlinear capacitors. An RLC circuit realized by such capacitors present chaotic solutions. We propose to study the nonlinear behavior by linearization in the sense of least squares.**

***Index Terms*—Model Landau, phase transition, capacitor nonlinear bifurcation optimal linearization,**

I. Introduction

The study of nonlinear systems has led physicists to the discovery of a large number of systems with complex behaviours and strange. These behaviours are similar to random behaviour although the systems are nonlinear deterministic have been explored in mathematics by new topological methods. The experiment, measurements and numerical simulations have shown that a large number of physical systems undergo bifurcation and chaos, beginning with mechanical systems the simplest: A system of multiple clocks with several equilibrium positions. The chaos was just noticed in the circuits nonlinear aero elastic systems, systems electro magneto mechanic, etc. ...

Indeed, a sequence of bifurcations in reducing turbulence was proposed by Landau. Rule and taken were the first to demonstrate that a strange attractor can be established from a finite sequence of bifurcation and creates turbulent models.

II. Nonlinear Capacitor Controlled by The Temperature

A nonlinear capacitor can be found for example in a varicap diode. However, there are dielectric can be controlled by temperature, allowing the realization of nonlinear capacitor.We present the thermodynamic potential of the dielectric as a function of P and T (which represents a size difference of atoms, may be the degree of order, or a size travel etc.. ...), If considers the phase transition at a given pressure, the function of the thermodynamic potential can be written :

$$\Phi(P,T,\zeta) = A(T) = a(T - T_c) \quad (1)$$

Where T_c is the temperature of transition phase and $a = \left.\frac{\partial A}{\partial T}\right|_{T=T_c}$, a constant in the phase transition model.

The dielectric energy per unit volume is written

$$G = \frac{A}{2}D^2 + \frac{C}{4}D^4 \quad (2)$$

Where C is a constant which depends on the temperature of transition phase, it is assumed positive. Since the electric field in material media checks

$$dG = EdD \quad (3)$$

the desired relation between D and E is written

$$E = AD + BD^3 \quad (4)$$

Equation (1) characterizing the material in the transition phase, can also describe the equilibrium positions.

The free energy of the dielectric has the following extrema

1) G presents a minimum: Where $T > T_c$ for $D = 0$.

2) G presents two minima: Where $T < T_c$ for $D = \pm\sqrt{-\frac{A}{B}}$.

3) G presents one maximum : When $T < T_c$, for $D = 0$.

We spend so continuously from the lowest symmetric state to the most symmetric state. (1)

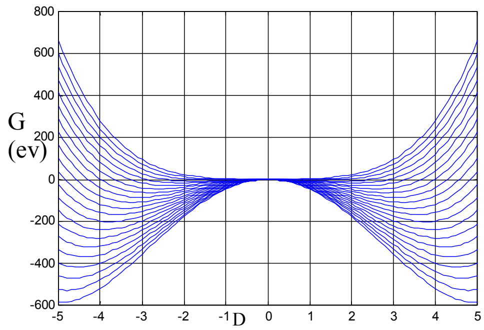


Fig.1: The energy of dielectric per unit volume for different temperatures.

Manuscript submitted, Dec 2009.

This work was supported the CNEPRU- MESRS Research program. MESRS..

T. Benouaz i is with the Laboratoire d'Automatique, B p 119 , Abu Bakr Belkaid University, Tlemcen, 13000, Algeria . (e-mail: T_benouaz@ mail-univ-tlemcen.dz).

F. Belkhouche , was with the Modeling laboratory, Department of physics,University of Tlemcen, B.p. 119, Tlemcen R.p

N. Ghouali,is was Laboratoire d'Automatique, B p 119 , Abu Bakr Belkaid University, Tlemcen, 13000, Algeria.

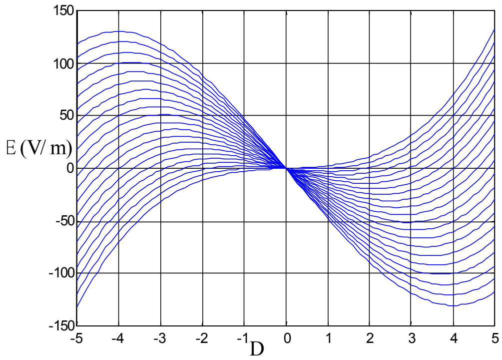
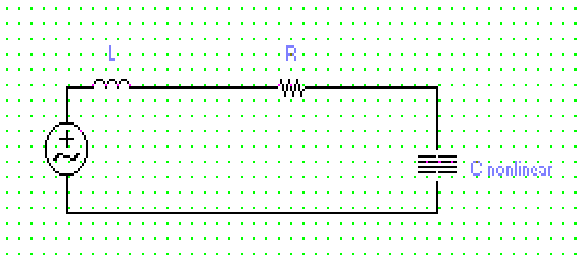


Fig.2: Electric Field for different Temperatures.

III. Circuits Made by a Non-Linear Capacitor

A. Non-linear RLC circuit

Over the past two decades research on the complex behaviour of nonlinear electronic circuits modelled by nonlinear equations has advanced rapidly. Chua's circuit occupies a special place, however, the nonlinearity in the circuit is resistive. The realization of an RLC circuit with nonlinear capacitor allows visualizing solutions and then understanding the dynamic behaviour of the circuit. The state equation governing the system is as follows



$$\ddot{D}+\frac{R}{L}\dot{D}+\frac{e}{LS}A_0(T-T_c)D+\frac{e}{LS}BD^3=\frac{U}{LS}\sin\omega t \cdot \quad (5)$$

Assume that the nonlinear capacitor is a capacitor which e is the thickness, S represents the surface. By solving the system (5) for :

$$R=1000\Omega, L=1.8\text{H}, S=0.0004\text{m}^2,$$
$$A_0=3.18e7, B=6.5e11,\ e=0.002\text{m},$$
$$Tc=23°.5$$

We note that :

1 - For a temperature lower than the temperature of phase transition solutions converge to an equilibrium point different from the origin while the origin is unstable.

2 - For a temperature above the transition temperature of phase solutions converge to an equilibrium point at the origin.

3 - At the temperature of phase transition we note that the solutions converge to a limit cycle.

We here merely to draw the solutions to the temperature phase transition figure (4) and Figure (5)

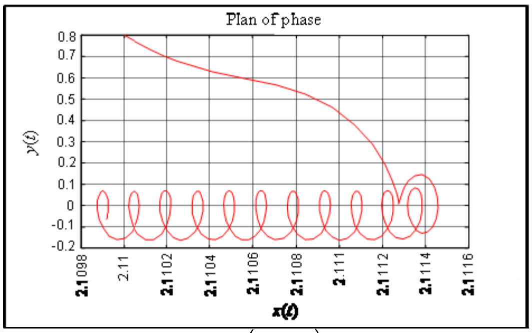


Fig.4: Phase Portrait of $(D,\dot{D})$ of the Temperature phase transition.

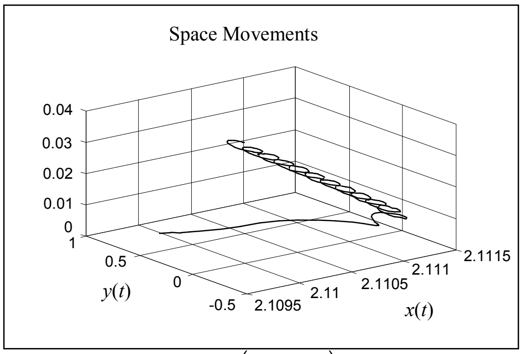


Fig.5: The space movement $(D,\dot{D},t)$ of the Temperature phase transition.

B. Non-linear circuit LC

The model of nonlinear LC circuit is as follows

$$\ddot{D}+\frac{e}{LS}A_0(T-T_c)D+\frac{e}{LS}BD^3=\frac{U}{LS}\sin\omega t \quad (6)$$

We note that this model has a similarity with the Duffing oscillator in some areas of the temperature, like the previous case, there are three regions:

1) The temperature is below the critical temperature: The circuit behaviour is similar to the Duffing oscillator. (Figure (6))

2) At the critical temperature (temperature of phase transition): the circuit has periodic solutions but controlled by the term non-self.

3) The temperature is above the critical temperature: the solutions have a center despite the existence of a term non-autonomous (Figure (7)).

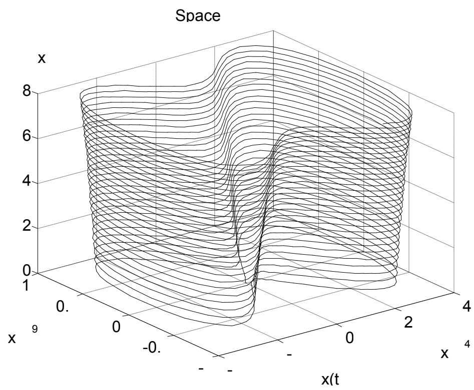


Figure 6: The space movement $(\text{D},\dot{\text{D}},\text{t})$ For $T>T_C$

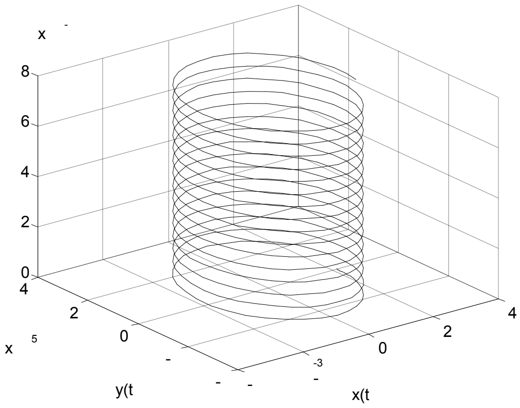


Figure 7: The space movement $\left(\mathrm{D}, \dot{\mathrm{D}}, \mathrm{t}\right)$ for $T < T_C$

C. Non-linear circuit RC

The model describing the circuit is as follows

$$\begin{cases} \dot{D} = -\dfrac{e}{SR} A_0 (T - T_c) D - \dfrac{e}{SR} B D^3 \\ D(0) = D_0 \end{cases} \tag{7}$$

We propose to study a model of the circuit system to the linearized free pitchfork bifurcation. The linearization method is the optimal derivation proposed by T. Benouaz and O. Arino [8] [9] [10] [11] [13] [14.. It is involved in the system (7), a system of form

$$\begin{cases} \dot{D} = AD \\ D(0) = D_0 \end{cases} \tag{8}$$

The procedure is based on minimizing the following functional (with $D(t) = x(t)$)

$$G(A) = \int_0^{+\infty} \left(\left\| F\left(x(t)\right) - Ax(t) \right\| \right) dt \tag{9}$$

Along a given solution, starting from a point x_0 and going to the equilibrium as time tends to infinity.

The derivation of (9) with respect to A gives

$$A = \left[\int_0^{+\infty} [F(x(t))][x(t)]^T dt \right] \times \left[\int_0^{+\infty} [x(t)][x(t)]^T dt \right]^{-1} \tag{10}$$

For the calculation of the matrix, the authors proposed an iterative method which can be summarized as follows (Assume that the spectrum of consecutive matrices is strictly negative real part)

1) $A_0 \leftarrow DF(x_0)$.

2) $j \leftarrow 1$.

3) $$A_j \leftarrow \left[\int_0^{+\infty} \left[F(e^{A_{j-1}t})\right]\left[e^{A_{j-1}t}\right]^T dt \right] \times \left[\int_0^{+\infty} \left[e^{A_{j-1}t}\right]\left[e^{A_{j-1}t}\right]^T dt \right]^{-1}$$

4) si $\left\| A_j - A_{j-1} \right\| > \varepsilon$.

$j \leftarrow j + 1$

Go To 3.

If No

Optimal derivative $\widetilde{A} \leftarrow A_j$

End.

We propose to study its behaviour when the temperature varies. The system has three equilibrium points, putting $\delta = \dfrac{A_0 (T - T_c)}{B}$ it obtains by the optimal derivative the following linear systems

Near the origin ($\dfrac{dD}{dt} = a_1 D = \left(\delta - \dfrac{D_0^2}{2} \right) D$

$$\tag{11}$$

In the vicinity of $D_{2eq} = \sqrt{\delta}$,

$$\frac{dD}{dt} = a_2 D = 2\left[-\delta - \frac{D_0^2}{4} - 2D_0\sqrt{\delta} \right] D \tag{12}$$

In the vicinity of $D_{3eq} = -\sqrt{\delta}$,

$$\frac{dD}{dt} = a_3 D = 2\left[-\delta - \frac{D_0^2}{4} + 2D_0\sqrt{\delta} \right] D \tag{13}$$

Indeed, to a temperature below the transition temperature phase the system has three equilibrium points, stability of each depends on the temperature, the problem is the value of phase transition, the method derivation provides an optimal balance unstable initially and then to conclude that the system undergoes a supercritical pitchfork bifurcation in the transition phase, Figures 8, 9 and 10 show the vector fields representing the electric induction state equilibrium. Before the temperature of phase transition temperature phase transition and after the temperature of phase transition.

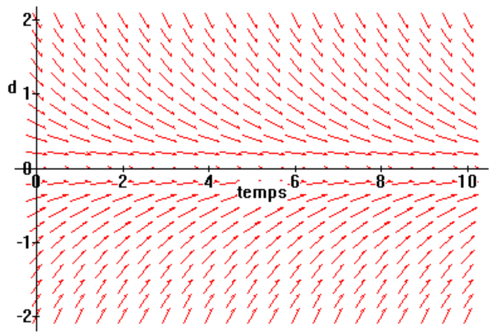


Figure.8: Vector field representing the electric induction near equilibrium at a temperature below the temperature of phase transition.

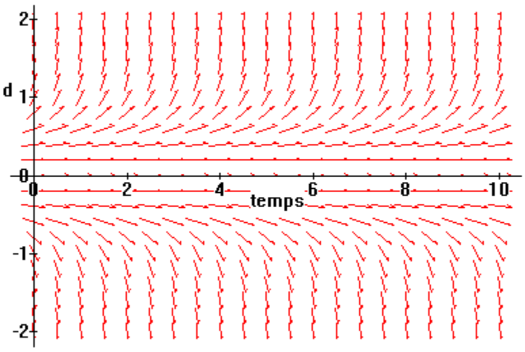


Figure 9: Vector field representing the electric induction near the equilibrium transition temperature phase.

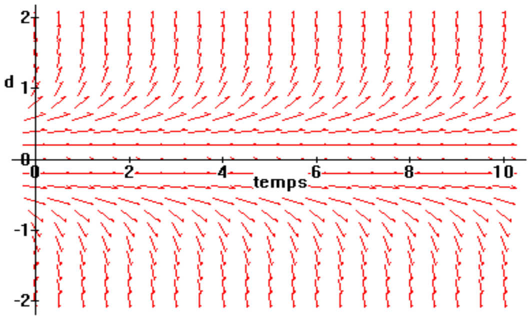


Figure 10: Vector field representing the electric induction near equilibrium at a temperature above the temperature of phase transition.

IV. Conclusion

The model of phase transition Landau helps explain the qualitative changes in the circuits made with a capacitor nonlinear behaviour of solutions can be varied form and undergoes several bifurcations. A sequence of bifurcation can be reduced to chaotic solutions.

A similar study may be to develop superconducting materials. In addition, the temperature of phase transition, the qualitative behaviour of solutions is ignored (since the state of the material is ignored a theoretical point of view). The linearized model proposed allows to know the nature of equilibrium in the value of bifurcation with great precision.

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Professor Tayeb Benouaz works as a Teaching, Department of Physics, Tlemcen university. Director of Post graduation of electronic physical modeling in the same university. His current research interest includes the computational physics, modeling and simulation of the nonlinear systems, applied mathematics. Director several research projects and has several publications in this field. The last publications: "Numerical simulation of nonlinear pulses propagation in a nonlinear optical directional coupler", International Journal of Physical Sciences Vol. 4 (9), pp. 505-513, September, 2009.ISSN 1992 - 1950 © 2009 Academic Journals. "On the Relationship between the Optimal Derivative and Asymptotic Stability ", African Diaspora Journal of Mathematics. Special Issue in Memory of Prof. Ibni Oumar Mahamat Saleh. Volume 8, Number 2, pp. 148–162 (2009) "Simulation and Experimental Studies of an Internal Thermal Insulation of two Pieces of Rooms Located in Ghardaia (Algeria)", International Journal of Advanced Computer Engineering , January- June 2009, Volume 2, issue 1. "Prediction model for the diffusion lengh in silicon-based solar cells", Journal of Semiconductors , Vol.30, N°.5, pp. 40001-1-40001-4, May 2009.

Professor Nouredinne Ghouali,is was the head of Laboratory of Automatique (2000-2009). The interesting work are Dynamical Systems, Power systems . . His current research interest includes the computational physics, modeling and simulation of the nonlinear systems, applied mathematics. Director several research projects and has several publications in this field.

Dr.F. Belkhouche , was in with the Modeling laboratory, Department of physics,University of Tlemcen,. His current research interest includes the computational physics, modeling and simulation of the nonlinear systems, applied mathematics., and has several publications in this field.