ISSN 1749-3889 (print), 1749-3897 (online)
International Journal of Nonlinear Science
Vol.9(2010) No.4,pp.480-485

Synthesis of Non Linear Electronic Systems Using Computational Procedure

T. Benouaz[1] , M.F. Bendahmane[1], A. Cheknane[2] *
[1]Laboratory of Electronic physics and Modeling University Abou Bakr Belkaïd, Tlemcen, Algeria
[2] Laboratory of Study and Development of the Materials Semiconductors and Dielectric, University Amar Telidji, Laghouat, Algeria

(Received 29 September 2009 , accepted 9 June 2010)

Abstract: The purpose of this paper is the use of the derived optimal for the synthesis of nonlinear systems. It may in particular act on their dynamic properties changing for example time of the transitional regime, or the first maximum of the solution.

Keywords: Nonlinear systems; Derivative optimal linearization; Synthesis

1 Introduction

The synthesis of a nonlinear system is to study its dynamic properties. For example, if you change the values of the dynamic characteristics of a system, electronics for example, can be varied [see 2, 3]: time for a transient regime, or the first maximum of the solution.
In the study of linear and nonlinear methods exist for the synthesis. This work is in line with the work of Jordan et al.. We will generalize the method used by Jordan in introducing the optimal derivation [5-12].

2 Theoretical Formalism of the Synthesis

2.1 Idea of the problem

Consider the following nonlinear system:

$$\begin{cases} \dfrac{dx}{dt} = F\left(x\left(t\right)\right) \\ x\left(0\right) = x_0, \end{cases} \tag{1}$$

$x \in I\!R^n$, F is a function defined in an open subset, with values in $I\!R^n$, under the following assumptions:

H1 $F\left(0\right) = 0$

H2 is γ Lipchitz continuous, of Lipchitz constant γ.

H3 The spectrum of $DF\left(x\right)$ is contained in the set $\{z : Re\ \ z < 0\}$ for every $x \neq 0$, in a neighborhood of 0, for which $DF\left(x\right)$ exists.

The goal is to seek to vary the dynamic characteristics of the system (1) and to match using synthesis, a nonlinear system of form

$$\begin{cases} \dfrac{dx}{dt} = L\left(x\left(t\right)\right) \\ x\left(0\right) = x_0 \end{cases} \tag{2}$$

corresponding to the corrected linear system

$$\begin{cases} \dfrac{dx}{dt} = \tilde{E}x \\ x\left(0\right) = x_0 \end{cases} \tag{3}$$

whose eigenvalues are S_i.

*Corresponding author. *E-mail address*: cheknanali@yahoo.com;Tel. : +213 29 93 17 91; Fax: +213 29 93 26 98.

IJNS.2010.06.30/376

2.2 Formalism

Assuming that

$$\left\{\begin{array}{l} \dfrac{dx}{dt} = \tilde{A}\,x \\ x(0) = x_0 \end{array}\right. \tag{4}$$

is the optimal linear system associated with the nonlinear problem (1) and that $\tilde{A}$ is the optimal matrix with eigenvalues $\lambda_i = 1, ..., n$.

To change the dynamics of nonlinear system (1), we must change the eigenvalues of the optimal matrix. For this, we must determine the matrix as K_B so that:

$$\frac{dx}{dt} = \left(\tilde{A} - K_B\right) x = \tilde{E}\,x \tag{5}$$

$\tilde{E}$ being the transition matrix of desired eigenvalues $S_i = 1, ..., n$.

K_B is determined as follows:

Let $S_i = 1, ..., n$,the desired eigenvalues.

To determine the K_B matrix, we must use the following steps:

- Calculate the eigenvalues of the optimal matrix.
- Define the eigenvectors of the $\tilde{A}$ as a matrix form denoted M.
- Define the diagonal matrix K whose elements K_{ij} are determined from

$$K_{ij} = \left\{\begin{array}{ll} \lambda_i - S_i & \text{if} \quad j = i \\ 0 & \text{if} \quad j \neq i \end{array}\right. . \tag{6}$$

- Using- the Jordan canonical form, and knowing that the matrix of eigenvectors M is a nonsingular matrix, the matrix relationship giving K_B is written as:

$$K_B = M \;\; K \;\; M^{-1}. \tag{7}$$

We verify that K_B admits eigenvalues.

Once the matrix K_B found, we calculate the transition matrix

$$\tilde{E} = \tilde{A} - K_B \tag{8}$$

of desired values $S_i = 1, ..., n$.

The new linear system is written

$$\left\{\begin{array}{l} \dfrac{dx}{dt} = \tilde{E}\,x \\ x(0) = x_0. \end{array}\right. \tag{9}$$

Once the linear system (9) determined, we will match it to a nonlinear system, knowing that the optimal matrix in this case is written as follows

$$\tilde{E} = \left[\int_0^{+\infty} \left[L\left(x\left(t\right)\right)\right]\left[x\left(t\right)\right]^T dt\right]\left[\Gamma\left(x\right)\right]^{-1}, \tag{10}$$

$L\left(x\left(t\right)\right)$ is the desired nonlinear system and $\Gamma\left(x\right) = \int_0^{+\infty}\left[x\left(t\right)\right]\left[x\left(t\right)\right]^T dt$.

This nonlinear system can be modeled as follows

$$\left\{\begin{array}{l} l_1 = \alpha_{11}g_1 + \alpha_{12}g_2 + \cdots + \alpha_{1j}g_n + R_1 \\ l_2 = \alpha_{21}g_{[n+1]} + \alpha_{22}g_{[n+2]} + \cdots + \alpha_{2j}g_{[n+j]} + R_2 \\ l_3 = \alpha_{31}g_{[2n+1]} + \alpha_{32}g_{[2n+2]} + \cdots + \alpha_{3j}g_{[2n+j]} + R_3 \\ \vdots \\ l_i = \alpha_{i1}g_{[(i-1)n+1]} + \alpha_{i2}g_{[(i-1)n+2]} + \cdots + \alpha_{ij}g_{[(i-1)n+j]} + R_i, \end{array}\right. \tag{11}$$

A more general way, it can be written as

$$\left\{\begin{array}{l} l_i = R_i + \sum_j^n = 1\; \alpha_{ij}g_{[(i-1)n+j]} \\ x(0) = x_0 \end{array}\right. \quad , \; i = 1, \cdots, n \tag{12}$$

with:

$g = h\left(x\right)$: is a function of chosen x, which may be linear or nonlinear and R_i are constants.

The unknowns are α_{ij}, we will seek to determine them by an identification term by term knowing the transition matrix $\tilde{E}$.

2.3 Synthesis Algorithm

Consider the following nonlinear system

$$\frac{dx}{dt} = F\left(x\left(t\right)\right),\ x\left(0\right) = x_0.$$

To perform the synthesis of this system, we must follow the following steps:

2.3.1 First Step

Determine the optimal linear system corresponding to the original nonlinear system by the method of the derived optimal. This system is therefore written

$$\begin{cases} \dfrac{dx}{dt} = \tilde{A}\ x \\ x\left(0\right) = x_0. \end{cases}$$

2.3.2 Second step

Change the dynamics of the optimal linear system and synthesize in order to obtain the desired dynamics for the nonlinear system. This change in dynamics occurs at the eigenvalues of the optimal matrix and will seek to find the transition matrix $\tilde{E}$.

2.3.3 Third step

The study of the stability of the new linear system

$$\begin{cases} \dfrac{dx}{dt} = \tilde{E}\ \ x \\ x\left(0\right) = x_0 \end{cases}$$

Corresponding to the desired nonlinear system.

2.3.4 Four step

Matching the new linear system (9) to a nonlinear system of form (12)

2.3.5 Fifth step

Calculate the values of the components of the new nonlinear system.
Once the nonlinear system (12) determined (by calculation of α_{ij}), we will be able to solve it numerically and compare the results with the desired results.

3 Application

3.1 Case of changing the time of the transient regime

In this section we will illustrate our study with the study of a physical system. In this example, we will vary the time of the phase of an electronic circuit.
The time variation of phase is possible due to the change that performs at the eigenvalues of the linear system obtained by the method of optimal derivative. If these eigenvalues are noted λ_i, and if we for example, varying the time of phase then the new eigenvalues S_i of the new linear system written

$$S_i = \frac{\lambda_i}{\eta},\ \eta \in I\!N \tag{13}$$

where we want to extend the time of the transient arrangements.

$$S_i = \lambda_i\ \eta,\ \eta \in I\!N \tag{14}$$

where we want to reduce the transient time. S_i are the eigenvalues of the transient matrix $\tilde{E}$. λ_i are the eigenvalues of the matrix optimal $\tilde{A}$. The determination of the transient matrix $\tilde{E}$ is made from eigenvalues S_i, following the calculation procedure described in (2). Once the matrix $\tilde{E}$ calculated, we determine the desired nonlinear system (12).
Now we shall illustrate this study using an example of an electronic circuit with two state variables, the nonlinearity arises from the presence of a diode non-linear (i, v).
Consider the following circuit.

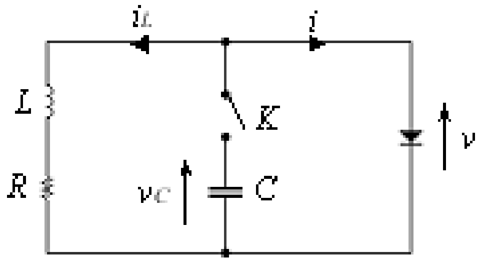

Figure 1: Nonlinear Electronic Circuit

There are two independent variables represented by the voltage drop v_C across the terminals of the capacity and the current through the coil i_L. Assume that the capacitor C is initially charged. We propose to study the behavior of the circuit after closing the switch K at $t = 0$.
When the voltage v is applied to a diode in the forward direction, the law of change of current passing through it is approximated by [1].

$$i = av + bv^2. \tag{15}$$

Applying Kirchhoff's mesh and nodes laws to the circuit lead us to equations of state

$$\begin{cases} \dfrac{dx}{dt} = -\dfrac{a}{C}x - \dfrac{b}{C}x^2 - \dfrac{1}{C}y \\ \dfrac{dy}{dt} = \dfrac{1}{L}x - \dfrac{R}{L}y, \end{cases} \tag{16}$$

with

$$\begin{cases} x = v_C \\ y = i_L. \end{cases} \tag{17}$$

Assume that the coefficients of the nonlinear characteristic of the line do not vary. For equation (12) we set

$$\begin{cases} g_1 = -x \\ g_2 = -bx^2 - y \\ g_3 = x \\ g_4 = y \\ R_1 = R_2 = 0 \end{cases} \tag{18}$$

we obtain

$$\begin{cases} l_1 = \alpha_{11}g_1 + \alpha_{12}g_2 + R_1 = -\alpha_{11}x - \alpha_{12}\left(bx^2 + y\right) \\ l_2 = \alpha_{21}g_3 + \alpha_{22}g_4 + R_2 = \alpha_{21}\,x + \alpha_{22}\,y. \end{cases} \tag{19}$$

For example, the circuit represented by the Fig.1 we apply the procedure described in the summary section (2), will allow us to determine the coefficients α_{ij}.
The values of circuit components are given by

$$\begin{array}{ll} R = 100\,\Omega & a = 3.5 \times 10^{-3}\,A/V \\ C = 5 \times 10^{-6}\ F & b = 10^{-2}\,A/V^2 \\ L = 0.5 \times 10^{-3}\,H\ . & \end{array} \tag{20}$$

The system (16) becomes

$$\begin{cases} \dfrac{dx}{dt} = -700\,x - 2 \times 10^3\,x^2 - 2 \times 10^5\,y \\ \dfrac{dy}{dt} = 2 \times 10^3\,x - 2 \times 10^5\,y \end{cases} , \qquad (x_0, y_0) = (5, 0)\,, \tag{21}$$

At the 4^{th} iteration, the calculation procedure gives us (with the desired accuracy $\varepsilon = 10^{-6}$)

$$\tilde{A} = \begin{bmatrix} -1.330 \times 10^4 & 4.338 \times 10^5 \\ 2 \times 10^3 & -2 \times 10^5 \end{bmatrix}. \tag{22}$$

This eigenvalues optimal matrix are

$$\begin{aligned} \lambda_1 &= -8.768 \times 10^3 \\ \lambda_2 &= -2.045 \times 10^5. \end{aligned} \tag{23}$$

The goal in this example is to extend the time of the transitional arrangements of the circuit studied. For this, we take the values of S_1 and S_2 of the transition matrix $\tilde{E}$ as follows

$$\begin{aligned} S_1 &= \frac{\lambda_1}{4} = -2.192 \times 10^3 \\ S_2 &= \frac{\lambda_2}{4} = -5.113 \times 10^4. \end{aligned} \tag{24}$$

From these eigenvalues, we can deduce the transition matrix following the calculation procedure outlined in Section (2) and we obtain

$$\tilde{E} = \begin{bmatrix} -3.325 \times 10^3 & 1.0845 \times 10^5 \\ 0.5 \times 10^3 & -0.5 \times 10^5 \end{bmatrix}. \tag{25}$$

The transition matrix $\tilde{E}$ admits eigenvalues initially selected S_1 and S_2. Now, knowing this transition matrix, we identify the coefficients α_{ij} are the unknowns in the expression of the desired nonlinear system (19).
After calculations we arrive to the following results

$$\begin{aligned} &\alpha_{11} = 1.1397 \times 10^2 \quad &\alpha_{12} = 5.1 \times 10^2 \\ &\alpha_{21} = 0.5 \times 10^3 \quad &\alpha_{22} = -0.5 \times 10^5. \end{aligned} \tag{26}$$

The nonlinear system (19) becomes

$$\begin{cases} l_1 = -1.1397 \times 10^2 x - 5.1 \times 10^4 \left(10^{-2} x^2 + y\right) \\ l_2 = 0.5 \times 10^3 x - 0.5 \times 10^5 y \end{cases} \qquad (x_0, y_0) = (5, 0), \tag{27}$$

and we deduce the values of elements of the new nonlinear system

$$\begin{aligned} \alpha_{11} &= \frac{a}{C} \\ \alpha_{12} &= \frac{1}{C} \Longrightarrow C = 1.9607 \times 10^{-5}\, F \\ \alpha_{21} &= \frac{1}{L} \Longrightarrow L = 2 \times 10^{-3}\, H \\ \alpha_{22} &= -\frac{R}{L} \Longrightarrow R = 100\, \Omega\,. \end{aligned} \tag{28}$$

In Figures (2) and Figure (3), curve (1) corresponds to solving the original nonlinear system (21), curve (2) to the solution of nonlinear system desired (27) and curve (3) the solution corresponding to the linear system (25).

We remark from the figures that there is a change in the time of the transient regime, since it can clearly be seen that the graph of the nonlinear system (27) converges slowly to a steady state compared to the graph of the original nonlinear system (21).

4 Conclusion

Through these two examples, we have show the importance of studying the synthesis of a nonlinear system using the method of the derived optimal. This is confirmed by the results, since we could highlight one of the main goals of the synthesis, i.e.: varying the time of the transient regime.
Therefore we believe that, given the properties [5] presented the optimal derivation; the study of the synthesis can be applied to several classes of nonlinear problems.

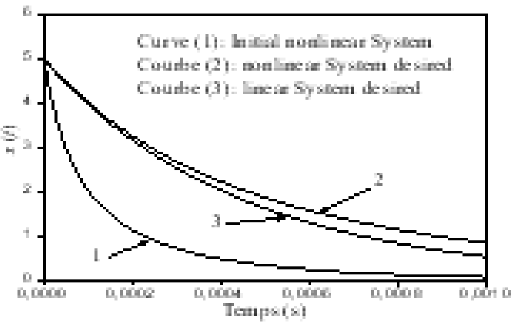

Figure 2: Variation of the solution x (t) as a function of time for the initial conditions $(x0, y0) = (5, 0)$.

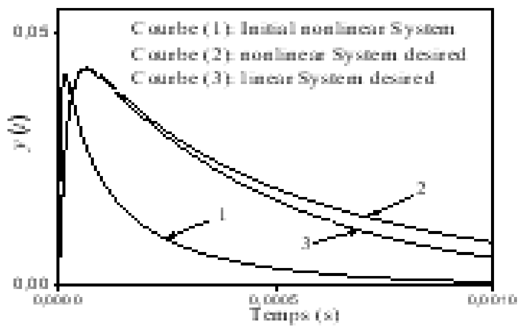

Figure 3: Variation of the solution y (t) as a function of time for the initial conditions $(x0, y0) = (5, 0)$.

References

[1] J. Cuningham, Introduction to Non-Linear Analysis, Mc-Graw.Hill, New-York, (1958).

[2] A.Fossard, Commande des Systèmes Multidimensionnels, Dunod, (1972).

[3] A. Volovoich, Linear Multivariable Systems, New-York Springer, (1974).

[4] A. Z. Bensenane, Linéarisation Optimale des Equations d'Etat Non Linéaires, Thèse de Magister, Tlemcen, (1986).

[5] T. Benouaz, Contribution a l'Approximation et la Synthèse de la Stabilité d'une Equation Différentielle Ordinaire Non Linéaire, Thèse de Doctorat d'Etat, Tlemcen, (1996).

[6] T. Benouaz, O. Arino, Existence, Unicité et Convergence de l'Approximation au Sens des Moindres Carrés d'une Equation Différentielle Ordinaire Non Linéaire, 94/14 Université de Pau, CNRS URA 1204, (1994).

[7] T. Benouaz, O. Arino, Relation Entre l'Approximation Optimale et la Stabilité Asymptotique, Publications de l'U.A, CNRS 1204, N°95/10, (1995).

[8] T. Benouaz, Optimal Derivative of a Non-Linear Ordinary Differential Equation, Equadiff 99, International Conference on Differential equations, Berlin (Germany) 1-7 August, Vol.2, pp.1404-1407, World scientific Publishing Co. Pte. Ltd, (2000).

[9] T. Benouaz, O. Arino, Optimal Approximation of the Initial Value Problem, Computers Math. Applic., Vol.36, N°1, pp.21-32, (1998).

[10] T. Benouaz, O. Arino, Determination of the Stability of a Nonlinear Ordinary Differential Equation by Least Square Approximation. Computational Procedure, Appl. Math. and Comp sci., N°1, Vol.5, pp.33-48, (1995).

[11] T. Benouaz, Lyaponov Function Generated by Least Square Approximation, Deuxième conférence Maghrébine sur l'Automatique, Tlemcen (Algeria) 3-5 December, Vol.1 pp.73-75, (1996).

[12] T. Benouaz, O. Arino, Least Square Approximation of a Non-Linear Ordinary Differential Equation, Computers Math. Applic., Vol.31, N°8, pp.69-84, (1996).

[13] M.F. Bendahmane, Contribution à l'Etude des Systèmes Non Linéaires avec Excitation, Thèse de Magister, Tlemcen (Algeria), (2000).