



## Study of additional heating Systems by Electron cyclotron and Alfvén waves in Tokamak Machine

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**GJSFR-A Classification** : 020201



STUDY OF ADDITIONAL HEATING SYSTEMS BY ELECTRON CYCLOTRON AND ALFVEN WAVES IN TOKAMAK MACHINE

*Strictly as per the compliance and regulations of :*



# Study of additional heating Systems by Electron cyclotron and Alfvén waves in Tokamak Machine

Naima Ghoutia Sabri<sup>α</sup>, Tayeb Benouaz<sup>Ω</sup>

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## 1. INTRODUCTION

Energy is essential to whole life; is a major scientific and strategic challenge to discover a new method of energy production that has an impact as low as possible on health, and the environment and the overall functioning of the planet, with sufficient energy to several million years. Energy produced from thermonuclear fusion reactions had been known for some decades in the sun and stars, is likely safe and doesn't produce greenhouse gas emissions and its radioactive wastes is less expensive to manage. These reactions require special conditions of temperature (100 million degrees) and pressure. In this case, the more promoter configuration to realize them is tokamak which is a machine governed by Lawson criterion (Sabri. N.G, 2010), ( $n$  density,  $T$  temperature and  $t_E$  confinement time) and to achieve these high temperatures, it is necessary to heat the plasma.

The ohmic regime is the first natural heating mechanism. Unfortunately, this effect is proportional to

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the resistance of the plasma which tends to collapse when the temperature increases. We therefore use additional heating systems. Radio-frequency heating (Wang<sup>†</sup>.S and Tang. J, 2004) is one of important of these systems. It is based on the phenomenon of wave-particle resonance where the waves can be transferred their energy to the charged particles in the plasma, which in turn collide with other plasma particles, thus increasing the temperature of the bulk plasma.

The injection of electron-cyclotron (EC) waves is nowadays a well-established method for coupling energy to plasma electrons in modern fusion devices (Harvey. R.W. and al., 1996, Mandrin. P., 1999), with primary applications the plasma heating ECRH (Electron Cyclotron Resonance Heating), (Arnoux. G., 2005) and the generation of non-inductive current drive ECCD (Electron Cyclotron Current Drive), (Dumont. R., 2001, Nikkola .P, 2004).

In this sense, the ECRH studies are formally split in the experiments involving the injection of EC waves on the one hand, and on the other in the theoretical investigations related to the propagation (Fontanesi. M and Bernabei. P, 1971) and absorption (Orefice. A, 1988), of the radiation. With respect to the theory, it is very important to have a quantitative model for the way the wave propagates and is absorbed inside the plasma (Bornaciti. M, 1982), as well as for the effects the resonant electrons have on the wave. The cold plasma model is used to describe the propagation (Stix. T.H, 1962), and the absorption is described with kinetic model (Rönmark. K. G., 1985).

The Alfvén wave is a fundamental electromagnetic oscillation in magnetically confined plasmas. Alfvén waves can be either excited spontaneously by instabilities or driven by external sources. It is also believed that Alfvén waves play a crucial role in the heating of bulk plasmas in both magnetic fusion devices and the solar corona. The Alfvén waves band are divided into slow shear Alfvén wave (SW) (Appert, 1986) and the fast compressional Alfvén wave (FW), (Cross. R. C and Lehane. J. A., 1967).

Heating plasma by resonant absorption of Alfvén waves is a technique that combines low-frequency conventional technology and low cost of installed capacity. The TCA Tokamak, acronym of heating in tokamak Alfvén wave. The TCA/Sw refers to

the circular section tokamak of CRPP (Center for Research in Plasma Physics, Switzerland) with the main objective is to investigate the possibility of plasma heating by dissipation of Alfvén waves (Cheethan 1980, TCA Team 1985, Chambrier. A. D., 1987) and TCA/Br refers to the Brazilian tokamak with Alfvén wave heating (Elfimov. A. G. and al. 1995, Ruchlto. L and al, 1994) is the largest machine and the most powerful and best equipped in diagnostics which provided the most detailed results on the spectrum and heating by Alfvén waves. Its purpose is to study the excitation and absorption Alfvén waves in plasma (Hasegawa, 1975, 1982), showing the usefulness of these waves in the additional heating.

In this paper, we examine in some depth, two types of plasma additional heating systems in a tokamak machine. The emphasis is on electron cyclotron heating. First, we briefly come back to the main non-collisional heating mechanisms and to the particular features of the quasilinear theory absorption in the electron cyclotron range of frequencies (ECRF). Then, the Alfvén wave heating is covered more briefly. Where applicable, the prospects for ITER are commented.

## II. WAVE PROPAGATION

### a) Cold Plasma

In this approximation the plasma pressure is assumed very small compared to the magnetic pressure  $\beta \ll 1$ . Where The parameter  $\beta$  represents the ratio of thermal pressure (kinetic)  $p = nk_B T$  and the magnetic pressure  $B^2/2\mu_0$ . With  $k_B$  is the Boltzmann constant;  $T$  and  $n$  are respectively the temperature and the density of electrons.  $B$  is the magnetic field and  $\mu_0$  the magnetic permeability in vacuum. In this case the thermal motion of electrons may be negligible in terms of oscillations of the wave  $v_\phi \gg v_{th}$  where  $v_\phi$  is the phase velocity of the wave and  $v_{th}$  is the thermal velocity of electrons and the Larmor radius is small compared to the wavelength (Bertrand. P, 2004). The relation between  $\vec{j}$  and  $\vec{E}$  can be written as

$$\vec{j}(\vec{k}, \omega) = \bar{\sigma}(\vec{k}, \omega) \cdot \vec{E}(\vec{k}, \omega) \tag{1}$$

Where  $\vec{k}$  is the wave vector,  $\bar{\sigma}$  is the conductivity of the plasma that is a tensor in case of anisotropic plasma. Considering plane wave solutions of Maxwell's equations, such as fluctuating quantities vary as  $\exp(i(\vec{k} \cdot \vec{r} - \omega t))$ . In Fourier space, we can find from the Maxwell's equations a wave equation of the form (Moncuque. M, 2001):

$$k^2 \vec{E} - \vec{k}(\vec{k} \cdot \vec{E}) - \left(\frac{\omega^2}{c^2}\right) \vec{D} = 0 \tag{2}$$

Where  $\vec{D} = \bar{K} \vec{E}$  is the electrical induction vector,  $\bar{K}$  is the dielectric tensor (permittivity),  $\vec{E}$  is the vector of wave electric field. In the cold plasma approximation, the dielectric tensor  $\bar{K}$  can be written in the following matrix form (Sabri. N.G, 2010, Moncuque. M, 2001) :

$$\bar{K} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix} \tag{3}$$

Where in the domain of electron cyclotron wave frequency ( $\omega \gg \omega_{ci}, \omega_{pi}$ ), S, D and P are given by

$$S = 1 - \frac{\omega_{pe}^2}{(\omega^2 - \omega_{ce}^2)} \tag{4}$$

$$D = -i \frac{\omega_{ce}}{\omega} \frac{\omega_{pe}^2}{(\omega^2 - \omega_{ce}^2)} \tag{5}$$

$$P = 1 - \frac{\omega_p^2}{\omega^2} \tag{6}$$

$$R = 1 - \frac{\omega_p^2}{\omega(\omega + \omega_c)} ; L = 1 - \frac{\omega_p^2}{\omega(\omega - \omega_c)} \tag{7}$$

With  $\omega_{pe}^2 = \frac{4\pi n_e e^2}{m_e}$ ,  $\omega_{ce} = \frac{eB_0}{m_e c}$ . Where  $n_e$  is the electron density,  $-e$  the electron charge and  $m_e$  its mass.

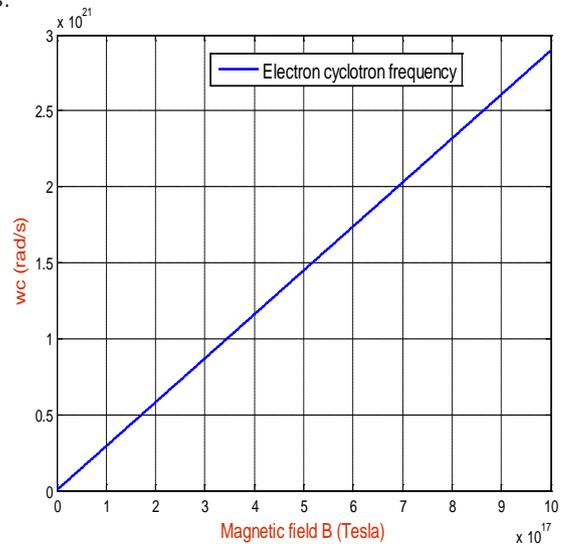
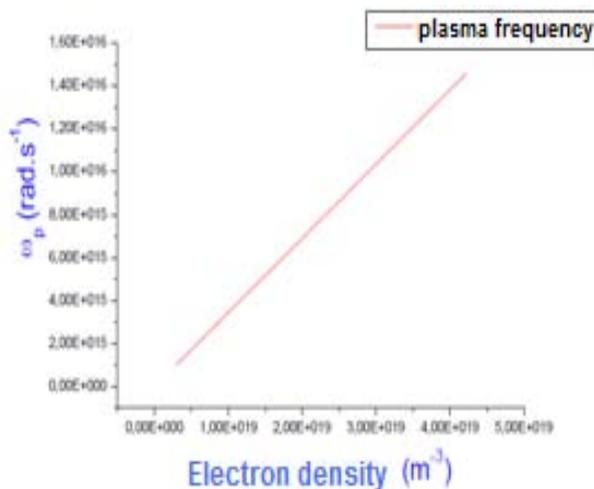


Fig.1 : Description of above image : the figure describes in (a) the plasma frequency as a function of density and in (b) the electron cyclotron frequency as a function of .

As the refractive index  $\vec{N}$  is written  $\vec{N} = \frac{\omega}{c} \vec{k}$ ; the equation (2) becomes  $\vec{M}_{k,\omega} \vec{E} = \vec{N} \wedge \vec{N} \wedge \vec{E} + \vec{K} \cdot \vec{E} = 0$  and the nontrivial solutions are obtained for:  $\det(\vec{M}_{k,\omega}) = 0$ , such as  $\vec{M}_{k,\omega}$  is a matrix representing the operator  $(\vec{k} \wedge \vec{k} \wedge \vec{\square} + \frac{\omega^2}{c^2} \vec{K} \cdot \vec{\square})$ . So we can write:

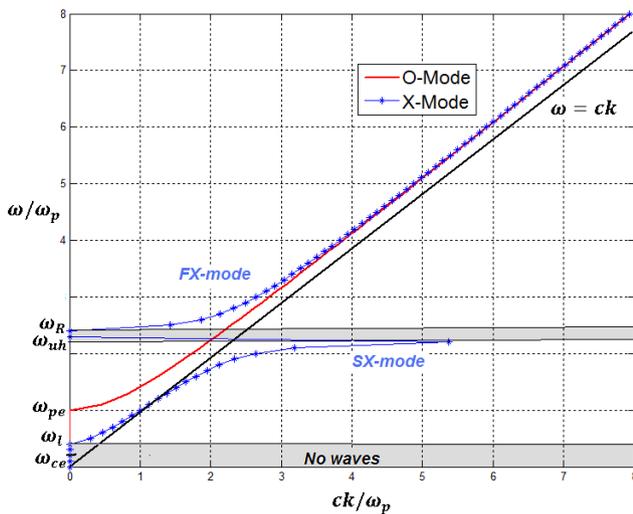
$$\begin{pmatrix} S - N^2 \cos^2 \theta & -iD & N^2 \cos^2 \theta \sin^2 \theta \\ iD & S - N^2 & 0 \\ N^2 \cos^2 \theta \sin^2 \theta & 0 & P - N^2 \sin^2 \theta \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0 \tag{8}$$

The following system may take the form of the dispersion equation as follows:

$$AN^4 + BN^2 + C = 0 \tag{9}$$

With  $A = S \sin^2 \theta + P \cos^2 \theta$ ,  $B = RL \sin^2 \theta + PS(1 + \cos^2 \theta)$  and  $C = PRL$ .

To generate current, the power of the electron cyclotron wave must be effectively absorbed by the plasma. However, the quality of the interaction depends on the state of polarization of this wave. It is useful, in this frequency range, using a proper mode (ordinary or extraordinary), chosen according to the plasma conditions, and assume that it propagates up the resonance without modification. In the case of perpendicular propagation to magnetic field ( $N_{||} = 0$ ).



We obtain two solutions of equation (9) for the perpendicular refractive index, which can be written:

$$N_0^2 = P = 1 - \frac{\omega_p^2}{\omega^2} \tag{10}$$

$$N_X^2 = \frac{S^2 - D^2}{S} = 1 - \frac{\omega_{pe}^2}{\omega^2} \frac{(\omega^2 - \omega_{pe}^2)}{(\omega^2 - \omega_{pe}^2 - \omega_{ce}^2)} \tag{11}$$

These electromagnetic solutions are well known by the names of ordinary mode (O) and extraordinary (X), (Sabri, N.G & Benouaz T, 2011).

- The ordinary mode (O): The electric field is parallel to the confining magnetic field and transverse ( $\vec{E} \perp \vec{k}$ ). This mode does not have any resonance and propagate for  $\omega > \omega_{pe}$  because of the cut-off.
- The extraordinary mode (X): The electric field is elliptically polarized in the perpendicular plane to  $\vec{B}_0$ . This mode has two cut-offs and two resonances.

According to the phase velocity  $\omega/k$ , there are two modes X, fast (F) and slow (S) as it is shown in Figure 2 and in Figure 3. This figure shows the dispersion relations of ordinary (O) and extraordinary (X) fast (F) and slow (S) waves propagating across the magnetic field.

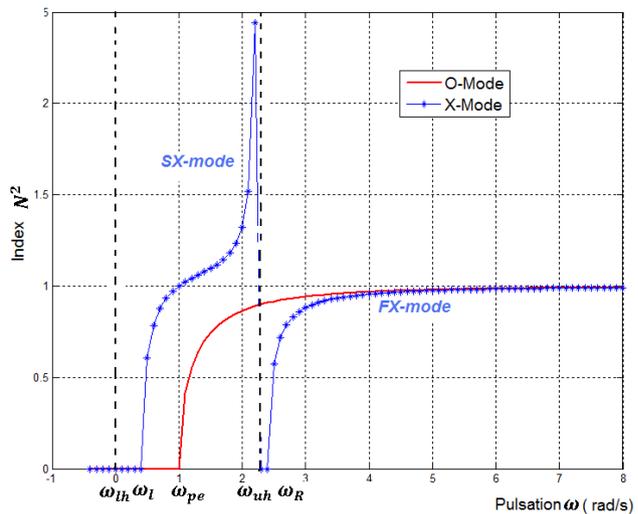


Fig.2: (a) The dispersion diagram (b)  $N^2 = f(\omega)$  for perpendicular propagation

The electron temperature, coupled with their average speed, such as

$$k_{\perp} \rho_L = N_{\perp} n \sqrt{k_B T_e / (m_e c^2)} \tag{12}$$

Where  $N_{\perp} = k_{\perp} c / \omega$  is the refractive index of the wave in plasma. The two branches of propagation (ordinary and extraordinary) appear and we can see that the ordinary mode propagates for frequencies such that  $\omega > \omega_{pe}$ . The extraordinary mode is propagative for

$\omega_L < \omega < \omega_{uh}$ , evanescent for  $\omega_{uh} < \omega < \omega_R$ . It becomes propagative when  $\omega > \omega_R$ . With  $\omega_R, \omega_L$  are the cutoff frequencies of the X mode, called right and left modes, defined by:

$$\omega_{R,L} = \frac{1}{2} \left[ \mp \omega_c + (\omega_c^2 + 4\omega_p^2)^{1/2} \right] \tag{13}$$

The X mode has a cold resonance ( $N_{\perp} \rightarrow \infty$ ), given by:

$$\omega_{uh} = \sqrt{\omega_c^2 + \omega_p^2} \tag{14}$$

This resonance is called upper hybrid (UH) is not available if  $\omega > \omega_c$ . There is also a lower hybrid resonance (Swanson. D.G, 1989), it is well below the electron cyclotron frequency domain and therefore not interferes here.

b) *Electron Cyclotron Wave Absorption*

The cyclotron resonance is, in principle, an interaction between the wave and particle motion (see Figure 3).

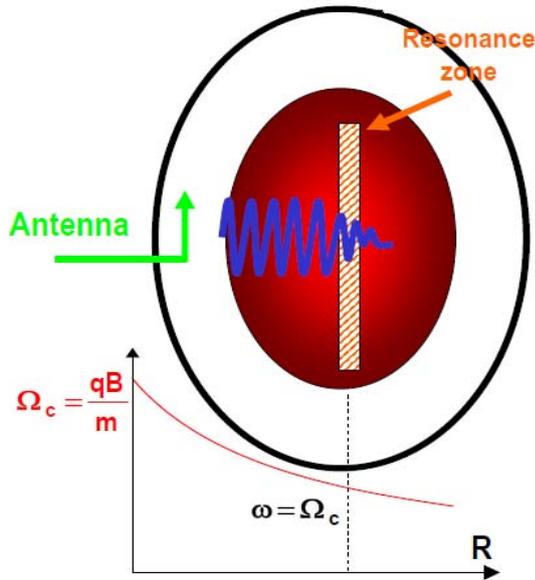


Fig.3 : The principle of EC heating.

In other words, it involves the microscopic structure of the plasma. We shall use the kinetic theory (as opposed to the fluid theory), to accurately reflect the phenomena occurring at the particle scale.

The hot plasma model under certain approximations, leads to a new expression of dielectric tensor that can be expressed by a correction of the type:

$$\bar{K}_{hot} = \bar{K}_{cold}(\omega, B_0, n_{e,0}) + \tilde{K}(\omega, B_0, n_{e,0}, T_{e,0}) \quad (15)$$

The hot correction  $\tilde{K}$  depends explicitly on the wave vector  $\vec{k}$  and the electron temperature at equilibrium,  $T_{e,0}$ . To calculate the elements of  $\bar{K}_{hot}$ , we start from the relativistic Vlasov equation (Arnoux. G, 2005, Dumont. R, 2001). In the relativistic formalism, the distribution function of electrons is written as  $f_e(\vec{r}, \vec{p}, t)$  with the relation  $\vec{p} = m_{e,0} \cdot \gamma \cdot \vec{v}$  where  $m_{e,0}$  is rest mass. The distribution function is solution of the relativistic Vlasov equation given by:

$$\frac{\partial f_e}{\partial t} + \frac{\vec{p}}{m_e \gamma} \frac{\partial f_e}{\partial \vec{r}} - e \left( \vec{E} + \frac{1}{m_e \gamma} \vec{p} \wedge \vec{B} \right) \frac{\partial f_e}{\partial \vec{p}} = 0 \quad (16)$$

Where  $m_e^2 = m_{e,0}^2 + (p/c)^2 = m_{e,0}^2 \gamma^2$  is the relativistic mass of the electron and  $\gamma = 1/\sqrt{1 - (v/c)^2}$

(Pochelon. A, 1994), the relativistic Lorentz factor,  $\gamma = 1$  for a non-relativistic plasma. The distribution function  $f_e$  is written as  $f_e(\vec{r}, \vec{p}, t) = f_{e,0}(\vec{p}) + f_{e,1}(\vec{r}, \vec{p}, t)$  the sum of two distribution functions  $f_{e,0}$  for equilibrium state and  $f_{e,1}$  for the perturbed state. Similarly to distribution function  $f_e$ , the magnetic and electric fields (Baea, Y.S, Namkung, W, 2004), can be written as  $\vec{B} = \vec{B}_0 + \vec{B}_1$  and  $\vec{E} = 0 + \vec{E}_1$ . A perturbed state of linearized Vlasov equation takes the form

$$\frac{\partial f_{e,1}}{\partial t} + \frac{\vec{p}}{m_e} \frac{\partial f_{e,1}}{\partial \vec{r}} + \frac{e}{m_e} (\vec{p} \wedge \vec{B}_0) \frac{\partial f_{e,1}}{\partial \vec{p}} = -e \left( \vec{E} + \frac{\vec{p} \wedge \vec{B}_1}{m_e} \right) \cdot \frac{\partial f_{e,0}}{\partial \vec{p}} \quad (17)$$

The integration of equation (17) gives the relativistic dielectric tensor:

$$K_{ij} = \delta_{ij} - \frac{\omega_p^2}{\omega^2} \frac{\mu^2}{2k_2(\mu)} \int_{-\infty}^{+\infty} d\bar{p}_{||} \int_0^{+\infty} \bar{p}_{\perp} d\bar{p}_{\perp} \frac{e^{-\mu y}}{\gamma} U \quad (18)$$

$$U = \sum_{n=-\infty}^{n=\infty} \frac{P_{i,j}^n(p_{\perp}, p_{||})}{\gamma - n \frac{\omega_{ce}}{\omega} - n_{||} \bar{p}_{||}} \quad (19)$$

Where  $\bar{p} = p/(m_{e,0}c) = \bar{p}_{\perp} + \bar{p}_{||}$ ,  $n_{||} = ck_{||}/\omega$  is the index refraction for parallel direction to  $\vec{B}_0$  and  $k_n(z)$  is the modified Bessel function of second kind (or McDonald function) of index  $n$  (here  $n = 2$ ) and argument  $z$ .

If we decompose respectively the dielectric tensor in hermitian and anti-hermitian parts as  $\bar{K} = \bar{K}_h + i\bar{K}_a$ . And if one decompose the hot correction  $\tilde{K}$  in real and imaginary part as  $\tilde{K} = \tilde{K}' + i\tilde{K}''$ . The expression (15) can be written:

$$\bar{K}_{hot} = \underbrace{\begin{pmatrix} S + \tilde{K}'_q & -i(D - \tilde{K}'_q) \\ i(D - \tilde{K}'_q) & S + \tilde{K}'_q \end{pmatrix}}_{hermitian} + i \underbrace{\begin{pmatrix} \tilde{K}''_q & i\tilde{K}''_q \\ -i\tilde{K}''_q & \tilde{K}''_q \end{pmatrix}}_{anti-hermitian}$$

It can be shown that the first hermitian part  $\bar{K}_h$  characterizes the propagation while the second anti-hermitian part  $\bar{K}_a$  characterizes the absorption (Swanson. D.G, 1989). If  $T_e \rightarrow 0$ , we obtain  $\bar{K}_a = 0$  and  $\bar{K}_h = \bar{K}_{cold}$ ; which justifies the use of the cold approximation to describe wave propagation (Brambilla. M, 1998).

c) *The Relation of Relativistic Resonance*

The relation of resonance is given by the relativistic resonance condition as follows:

$$\gamma - k_{||} v_{||} - n \frac{\omega_{c,0}}{\omega} = 0 \quad (21)$$

The term  $k_{||} v_{||}$  describes longitudinal Doppler shift [9], ( $k_{||} \neq 0$ ). The term  $n\omega_{ce}/\omega$  describes the gyration of the electron;  $n$  is the order of the harmonic

excited. This relation expresses the equality between the frequency of the wave and the relativistic cyclotron frequency of rotation corrected by the Doppler shift which caused by the electron parallel velocity. The energy of resonant electrons at  $\omega_{ce}$  and given  $n_{II}$  can be written as:

$$E = m_e c^2 (k_{II} v_{II} + n \frac{\omega_{ce}}{\omega} - 1) \tag{22}$$

An increase of the electron parallel velocity of the quantity  $\Delta v_{II}$  translates into a gain in elementary current  $\Delta j = -e \Delta v_{II}$ . Energy expense is increased by the electron  $\Delta E = m_e \cdot v_{II} \cdot \Delta v_{II}$ . So we deduce

$$\Delta j = e \frac{\Delta E}{m_e \cdot v_{II}} \tag{23}$$

This relation translates the generation of electron cyclotron current drive (ECCD) which is an important tool for current profile shaping in magnetically confined plasmas, thanks to the highly localized power deposition of the EC wave and the ease of external control of its deposition location.

d) Absorption Coefficient

We take the viewpoint of geometrical optics by considering a plane monochromatic wave type  $\vec{E}(\vec{r}, t) = \vec{E}(\vec{k}, \omega) \exp\{i[\vec{k} \cdot \vec{r} - \omega t]\}$  for which one trying to describe the dissipation by introducing the concept of absorption coefficient. For there to be absorption, it is necessary that  $k = k' + ik''$  avec the imaginary part of wave vector  $k'' = (\omega/c)N'' \neq 0$ . Then the absorption coefficient (Bornatici M. and al.,1983, Tsironis.C, Vlahos.L, 2006) is given by

$$\alpha = -2k'' \cdot \frac{\vec{v}_g}{v_g} \tag{24}$$

With  $\vec{v}_g = \frac{d\vec{r}}{dt}$  is the group velocity

For the explicit calculation of the absorption coefficient, we introduce a Another approach based on energy conservation, using the anti-Hermitian part of the dielectric tensor. Poynting's theorem (Sabri, N.G, Benouaz T.and Cheknane A., 2009) writes:

$$\frac{\partial W_{0,t}}{\partial t} + \vec{\nabla} \cdot \vec{S}_{0,t} = \frac{\partial}{\partial t} \frac{1}{2} \left( \frac{|\vec{B}_t|^2}{\mu_0} + \epsilon_0 |\vec{E}_t|^2 \right) + \frac{1}{\mu_0} \vec{\nabla} \cdot \text{Re}(\vec{E}_t \wedge \vec{B}_t) - \vec{j}_t \cdot \vec{E}_t \tag{25}$$

$\partial W_{0,t} / \partial t$ , contains respectively the magnetic  $|\vec{B}_t|^2 / (2\mu_0)$  and electrostatic  $\frac{1}{2} \epsilon_0 |\vec{E}_t|^2$  energies.  $\vec{S}_{0,t}$  is the instantaneous Poynting vector in vacuum describing the flow of electromagnetic energy. The source term,  $-\vec{j}_t \cdot \vec{E}_t$ , describes the interactions of the wave with the plasma. By performing the time average over a few periods of oscillations:  $\langle \vec{E}_t \rangle_t = \vec{E}_1(\vec{r}) \exp(i\vec{k} \cdot \vec{r})$ , and separating explicitly the parties hermitian and

antihermitienne of dielectric tensor introduced into the source term, we can be extracted from equation (25) the absorption coefficient:

$$\alpha = \frac{\epsilon_0 \omega \vec{E}_1^* \vec{K}_a \vec{E}_1}{|\vec{S}|} \tag{26}$$

Where  $\vec{E}_1^*$  is the complex conjugate of  $\vec{E}_1$  and  $\vec{S} = \vec{S}_0 + \vec{Q}_s$  with

$$\vec{S}_0 = \frac{1}{4\mu_0} \text{Re}(\vec{E}_1^* \wedge \vec{B}_1 + \vec{E}_1 \wedge \vec{B}_1^*) \tag{27}$$

$$\vec{Q}_s = -\frac{1}{4} \epsilon_0 \omega \vec{E}_1^* \frac{\partial \vec{K}_h}{\partial k} \cdot \vec{E}_1 \tag{28}$$

And for wave polarized in X-mode, the absorbed power density is given by the numerator of (26) as:

$$D = \epsilon_0 \omega \vec{E}_1^* \vec{K}_a \vec{E}_1 = \alpha |\vec{S}| \tag{29}$$

A useful quantity is the optical depth  $\tau$  (Arnoux. G, 2005, Westerhof. E, 2006 and Mandrin. P, 1999). [2], [15], [16], which is defined as the integral of the absorption coefficient  $\alpha$  along the trajectory  $s$  of the wave:  $\tau = \int -\alpha ds$ . The total absorbed power  $P_{abs}$  in the plasma can then be written as

$$P_{abs} = P_{inj} (1 - \exp(-\tau)) \tag{30}$$

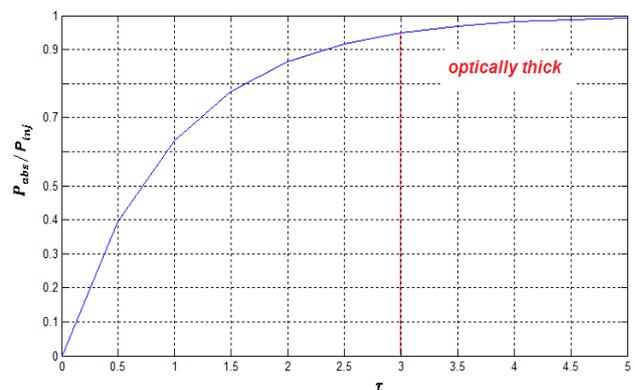


Fig.4 : The fraction of absorbed power as a function of optical depth  $\tau$ , (cas  $\tau > 3$ ).

We can see an illustration of the function  $P_{abs}/P_{inj}$  on the Figure 4 where we define that the plasma is optically thick when  $\tau > 3$ , i.e. when the fraction of absorbed power  $P_{abs}/P_{inj} > 95\%$ .

Table 1 present the optical depths of a plasma slab in which the magnetic field varies as  $B \sim 1/R$  and we obtained:

1. For the O-mode, the optical depth is given for perpendicular propagation and for all harmonics  $n \geq 1$ .

2. Similarly for the X-mode and the harmonics  $n \geq 2$ .
3. The optical depth for the fundamental harmonic  $n = 1$  of the X-mode is given for oblique propagation.

Table 1 : The optical depth of EC waves (Westerhof. E, 2006).

mode	expression
O-mode- $\perp$ $n \geq 1$	$\tau = \frac{\pi^2 n^{2(n-1)}}{2^{n-1}(n-1)!} N_0^{2n-1} \left(\frac{\omega_p}{\omega_c}\right)^2 \left(\frac{v_t}{c}\right)^{2n} \frac{R}{\lambda}$
X-mode- $\perp$ $n \geq 2$	$\tau = \frac{\pi^2 n^{2(n-1)}}{2^{n-1}(n-1)!} A_n \left(\frac{\omega_p}{\omega_c}\right)^2 \left(\frac{v_t}{c}\right)^{2(n-1)} \frac{R}{\lambda}$ With $A_n = N_X^{2n-3} \left(1 + \frac{(\omega_p/\omega_c)^2}{n(n^2-1-\omega_p^2/\omega_c^2)}\right)$
X-mode oblique $n = 1$	$\tau = \pi^2 N_X^5 \left(1 + \frac{\omega_p}{\omega_c}\right)^2 \left(\frac{\omega_c}{\omega_p}\right)^2 \left(\frac{v_t}{c}\right)^2 \cos^2 \theta \frac{R}{\lambda}$

In the table,  $v_{th} = (k_B T_e / m_e)^{1/2}$  is the thermal velocity of the electrons. In most current ECRH tokamak experiments either the fundamental O-mode or second harmonic X-mode are employed. Except near the edges of the plasma, optical depths of the order of one or significantly larger are generally achieved for both the fundamental O- and second harmonic X-mode resulting in complete single pass absorption.

e) *Electron Cyclotron Absorption in Tokamak Plasma*

In current fusion machines, the accessibility conditions usually require to inject the electronic cyclotron waves from low-field side. This imposes constraints on the polarization and the chosen mode from firstly of the propagation characteristics of ordinary and extraordinary modes and secondly from the absorption characteristics. So it is advantageous to use low-order harmonics of the interaction, to maximize absorption.

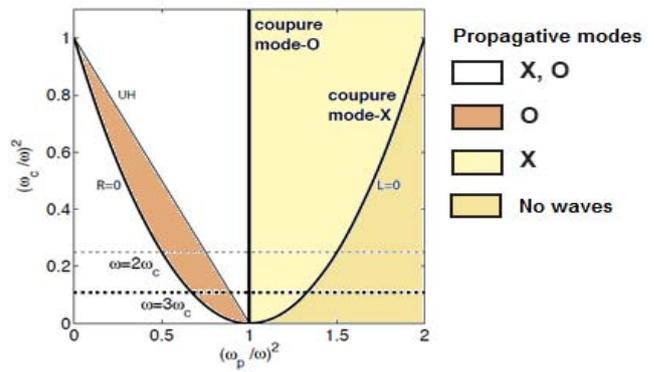
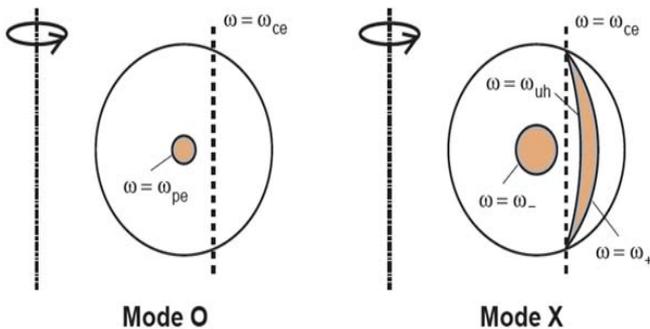


Fig.5 : (a) Typical cut-offs and resonances of a tokamak plasma in the case of perpendicular injection from the low-field side. Ordinary mode (left) and extraordinary mode (right); (b) CAM diagram.

The Figure 5 (a) shows the typical shapes of cut-offs right ( $\omega_R$ ), left ( $\omega_L$ ), and cut-off plasma  $\omega_{pe}$ , the high hybrid resonance  $\omega_{uh}$  and cyclotron frequency  $\omega_{ce}$  in the poloidal plane. A very synthetic way to represent this problem of choosing the mode and propagation is the CMA diagram, as is shown on the Figure 5(b), (Dumont. R, 2001).

### III. ALFVEN WAVES HEATING

a) *Alfven Waves Dispersion*

Branches of dispersion oblique propagation have a complicated expression because the continuation between  $\theta = \pi / 2$  and  $\theta = 0$ . In this case the wave propagates with a low frequency approximation checking the magnetohydrodynamic (MHD)  $\omega \ll \omega_{ci}, \omega_{pi}$ . The elements of dielectric tensor are given by:

$$S = 1 + \frac{\omega_{pi}^2}{\omega_{ci}^2 - \omega^2} + \frac{\omega_{pe}^2}{\omega_{pe}^2 - \omega^2} \approx 1 + \frac{\omega_{pi}^2}{\omega_{ci}^2} = 1 + \frac{c^2}{v_a^2} \quad (31)$$

$$D \approx \frac{i\omega}{\omega_{ci}} \frac{c^2}{v_a^2} \approx 0 \quad (32)$$

$$P \approx 1 - \frac{\omega_{pi}^2 + \omega_{pe}^2}{\omega^2} \approx 1 - \frac{c^2}{v_a^2} \frac{\omega_{ci} \omega_{ce}}{\omega^2} \approx -\frac{\omega_{pe}^2}{\omega^2} \gg 1$$

$$P \rightarrow \infty \quad (33)$$

Here, we used the quasi-neutral plasma, which is written  $\omega_{pe}^2 / \omega_{ce} = -\omega_{pi}^2 / \omega_{ci}$  and the system of eigenvalues (8) reduces to

$$\begin{cases} \left(-n^2 \cos^2 \theta + 1 + \frac{c^2}{v_a^2}\right) E_x = 0 \\ \left(-n^2 + 1 + \frac{c^2}{v_a^2}\right) E_y = 0 \\ (\infty) E_z = 0 \end{cases} \quad (34)$$

b) *Shear Alfvén wave (torsional Alfvén Wave)*

The first equation of system (34) gives the dispersion relation

$$n^2 \cos^2 \theta = 1 + \frac{c^2}{v_a^2} \tag{35}$$

It is fairly easy to show, from the definitions of the plasma and cyclotron frequencies that  $\frac{\omega_{pi}^2}{\omega_{ci}^2} = \frac{c^2}{v_a^2}$ . Here,  $\rho \approx nm_i$  is the plasma mass density, and

$$v_a = \sqrt{\frac{B_0^2}{\mu_0 \rho}} \tag{36}$$

is called the *Alfvén velocity*. Thus, the dispersion relations of the two low-frequency waves can be written

$$\omega \approx kv_a \cos \theta \equiv k_{||} v_a \tag{37}$$

With a phase velocity

$$v_\phi \approx v_a^2 \cos^2 \theta \tag{38}$$

It is interesting to note that the magnetic perturbation induces torsion of field lines and is therefore called *slow* or *shear* Alfvén wave (Dumont.R, France 2005 and Fitzpatrick.R , 2008); see Figure 8.

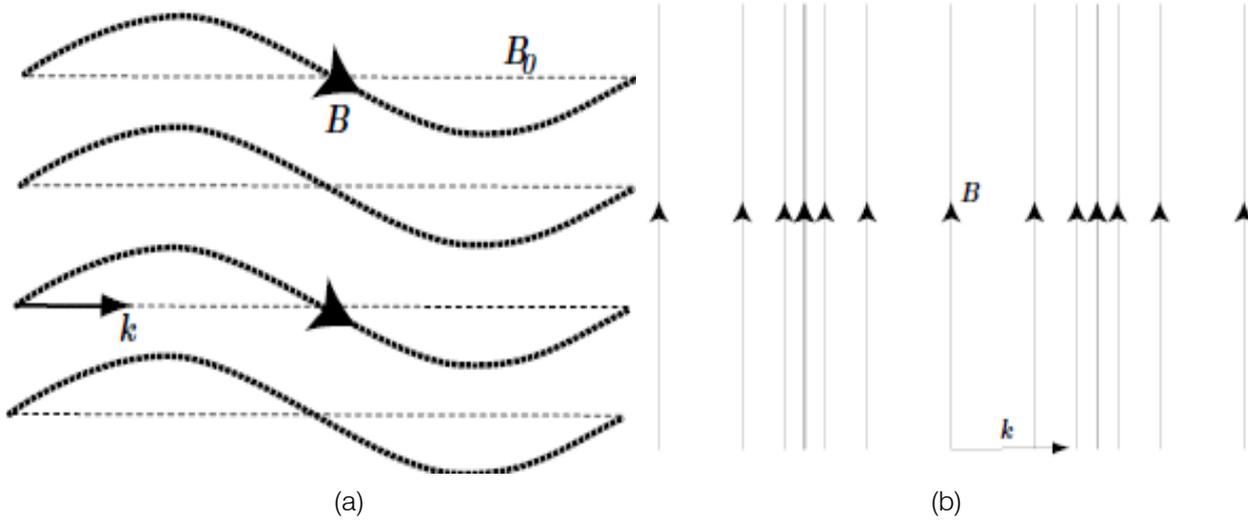


Fig.7 : Magnetic field perturbation associated with a (a) Shear-Alfvén wave; (b) Compressional Alfvén-wave.

c) *Principle of Alfvén wave heating*

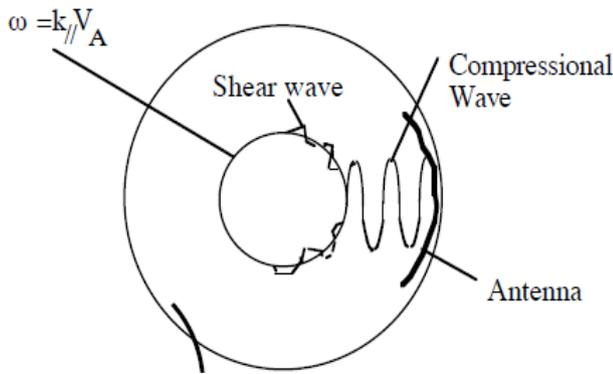
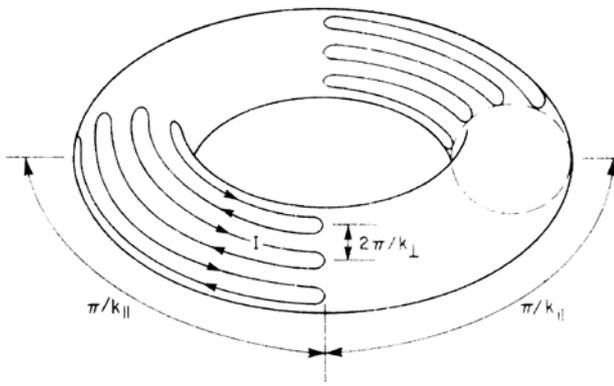


Fig.8 : The principle of Alfvén wave heating. Poloidal cross-section of the tokamak (Elfimov. A. G and al., 1995, Chambrier.A. D,1987).

The dispersion relation (35) implies that the shear Alfvén wave can propagate only along the field lines and in an inhomogeneous plasma there is only one surface, close to a magnetic surface, where for a given  $N_{||}$  this wave dispersion relation is satisfied. So, the shear Alfvén wave can propagate only on that surface, as shown on Figure 8, it is trapped on that surface.

Therefore, the idea is to launch from the outside the compressional Alfvén wave, which can propagate in all directions and reach the Alfvén resonance. Once the power is coupled to the shear wave by resonance absorption, it stays on the magnetic surface and dissipates there. The Figure 9 shows a schematic diagram of heating by the shear Alfvén wave resonance whose condition is

$$\omega^2(r) = \frac{k_{||}^2 v_a^2}{1 + k_{||}^2 v_a^2 / \omega_{ci}^2} \tag{42}$$



**Fig.9 :** Schematic diagram of the proposed setting of heating coil using shear Alfvén wave resonance (Hasagawa. A., Chen. L.,1974).

Note that the wavelength of the compressional wave is of the order of 1m. This means that, for 1m wide or narrower antennas, most of the wave spectrum will be evanescent with an evanescence length of the order of the antenna size (Westerhof. E, 2006). The inclusion of kinetic effects, such as electron and ion temperatures and finite electronic mass, changes the physical picture of processes. This gives rise to an electrostatic wave which propagates in radial direction close to resonant surface of (SW) that called kinetic Alfvén wave (Veron.D, 1978). In this case, the dissipation of the waves is attributed to Landau damping on electrons (Hasagawa. A., Chen. L.,1974). From the experimental point of view the most extensive experiments and analysis of Alfvén wave heating have been performed on the TCA tokamak (Veron.D, 1978).

#### IV. SUMMARY AND DISCUSSION

The application of EC waves to plasmas rests on a wide base of theoretical work which progressed from simple cold plasma models to hot plasma models with fully relativistic physics to quasilinear kinetic Vlasov models. In this case, all the information about the absorption of the EC wave in the inhomogeneous plasma, is finally expressed in terms of the relativistic dielectric tensor which characterizes the propagation with its hermitian part and the absorption with its anti-hermitian one. For a very low electron temperature  $T_e \rightarrow 0$ , the hermitian part of the tensor present the cold dielectric tensor which justifies the use of the cold plasma approximation to describe the wave propagation.

We generally use the cold plasma model to study Alfvén waves and especially to describe the damping of the compressional wave by local absorption of power at the position of the shear Alfvén wave resonance.

Although antenna coupling and general Alfvén wave behavior appeared to be in agreement with the

theory, generally speaking little plasma heating was observed while the main effect of the RF was a large density increase, sometimes interpreted as an increase in the particle confinement time. In view of these disappointing results there have been few attempts to apply Alfvén wave heating to large tokamaks and this method is usually not mentioned for the heating of ITER or reactors. However, there has been some renewed interest in this field as the conversion to the kinetic Alfvén wave may induce poloidal shear flows, and possibly to generate transport Barriers (Veron.D, 1978).

In contrast, electron cyclotron (EC) power has technological and physics advantages for heating and current drive (CD) in a tokamak reactor, and advances in source development make it credible for applications in the International Thermonuclear Experimental Reactor (ITER). Because this heating system (ECH) is a particularly *robust* heating scheme since the physics of wave propagation and absorption is well understood, there is total absorption for all plasma parameters foreseen in ITER.

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