Study of energy transfer by electron cyclotron resonance in tokamak plasma

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Abstract

A theoretical study of energy transfer by electron cyclotron resonance to tokamak plasma is presented. Then the predictions of linear theory including relativistic effects on the wave absorption are examined. Electron-cyclotron (EC) absorption in tokamak plasma is based on interaction between wave and electron cyclotron movement when the electron passes through a layer of resonance at a fixed frequency which depends on the magnetic field. This technique is the principle of additional heating (ECRH) and the generation of non-inductive current drive (ECCD) in modern fusion devices. The power absorbed depends on the optical depth which in turn depends on coefficient of absorption and the order of the excited harmonic for O-mode or X-mode.
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1. Introduction

The need to have a secure and clean supply of energy for our growing industrial civilization has led us to search for alternative supplies of energy. Energy produced from thermonuclear fusion reactions had been known for some decades in the sun and stars, is likely safe and don't produces greenhouse gas emissions and its radioactive wastes is less expensive to manage.

These reactions require special conditions of temperature (100 million degrees) and pressure. In this case, the more promoter configuration to realize them is tokamak which is a machine governed by Lawson

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criterion [1], $nTt_E \geq 5.10^{21} \, m^{-3}keVs$ and to achieve these high temperatures, it is necessary to heat the plasma. The ohmic regime is a primary natural mechanism of heating. Unfortunately, this effect is proportional to the resistance of the plasma which tends to collapse when the temperature increases. We therefore use additional heating systems. Radio-frequency heating is one of important of these systems. This phenomenon occurs if the waves have a particular frequency (the same as charged particles frequency), their energy can be transferred to the charged particles in the plasma, which in turn collide with other plasma particles, thus increasing the temperature of the bulk plasma.
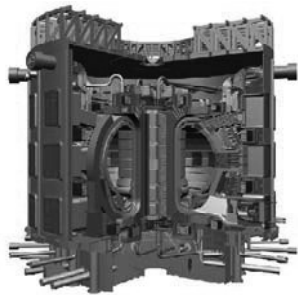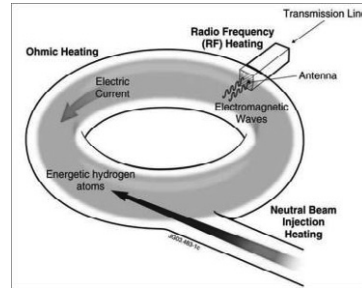

Fig. 1. (a) Tokamak machine- ITER;   (b) Heating methods

According the frequency range, there are three main types of radio-frequency heating [2]:
- The heating at the ion cyclotron frequency (ICF): a few tens of megahertz (MHz).
- The heating at hybrid frequency: a few gigahertzes (GHz).
- The heating at the electron cyclotron frequency (ECF): the hundreds of (GHz).

2. Electron Cyclotron Frequency

If the particle is an electron $q = -e$; its frequency of rotation ω_{ce} is called the electron cyclotron frequency given by $\omega_{ce} = 2\pi f_c = \frac{e\,B}{\gamma m_e}$. Where $\gamma = 1/\sqrt{1 - (v/c)^2}$ [3], the relativistic Lorentz factor, $\gamma = 1$ for a non-relativistic plasma (v << c).
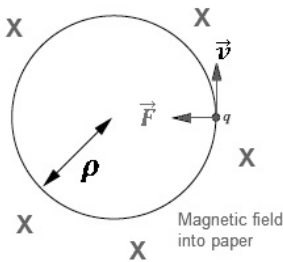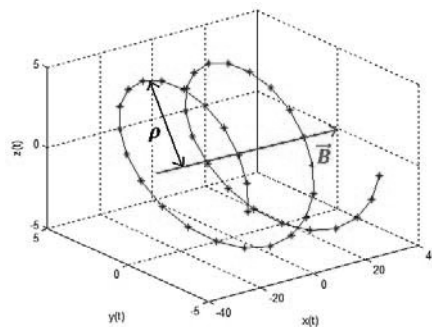

Fig. 2. (a) Effect of magnetic field on charged particle   (b) spiral trajectory of charged particle

The radius of the circular rotation is called Larmor radius given by

$$\rho = \frac{\gamma m_e v_\perp}{e\,B} \tag{1}$$

With $v_\perp$ is the component of velocity perpendicular to $\vec{B}$. Since the gyration of electrons is periodic, it emits radiation in a series of harmonics

$$\omega_{cen} = \frac{n\omega_{ce}}{1 - \frac{v_{II}}{c}\cos\theta} \tag{2}$$

Where n is the number of harmonics, v_{II} is the velocity component parallel to $\vec{B}$: The angle θ between the line of sight and the magnetic field $\vec{B}$. For $\theta \neq 90°$, we observe an oblique electron cyclotron emission. If $\theta = 90°$; equation (2) can be rewritten as $\omega_{cen} = n\omega_{ce} = 2\pi n f_c$.

2. Propagation and Dispersion Relation

To describe the propagation of electron cyclotron waves in plasma is generally used the cold plasma approximation [4]. In this approximation the plasma pressure is assumed very small compared to the magnetic pressure $\beta \ll 1$. In this case the thermal motion of electrons may be negligible in terms of oscillations of the wave $v_\varphi \gg v_{th}$ where v_φ is the wave phase velocity and v_{th} is a thermal velocity of electrons and the Larmor radius is small compared to the wavelength [5]. Considering plane wave solutions of Maxwell's equations, such as fluctuating quantities vary as $exp\,(i\,(\vec{k}.\vec{r} - \omega t))$. In Fourier space, we can find a wave equation of the form [6]:

$$k^2\vec{E} - \vec{k}(\vec{k}\vec{E}) - \left(\frac{\omega^2}{c^2}\right)\vec{D} = 0 \tag{3}$$

Where $\vec{k}$ is the wave vector, $\vec{D} = \overline{\overline{K}}\vec{E}$ is the electrical induction vector, $\overline{\overline{K}}$ is the dielectric tensor [1] [4], [7], $\vec{E}$ is the vector of wave electric field. If the refractive index is written as $\vec{N} = \frac{\omega}{c}\vec{k}$, the equation (3) can conduct to resolving the dispersion equation which may take the form :

$$AN^4 + BN^2 + C = 0 \tag{4}$$

With $A = S\sin^2\theta + P\cos^2\theta$, $B = RL\sin^2\theta + PS(1 + \cos^2\theta)$ and $C = PRL$. In the case of perpendicular propagation to magnetic field ($N_{II} = 0$). We obtain two solutions of equation (4) for the perpendicular refractive index, which can be written:

$$N_O^2 = P = 1 - \frac{\omega_p^2}{\omega^2}, \tag{5}$$

$$N_X^2 = \frac{S^2 - D^2}{S} = 1 - \frac{\omega_{pe}^2}{\omega^2}\frac{(\omega^2 - \omega_{pe}^2)}{(\omega^2 - \omega_{pe}^2 - \omega_{ce}^2)} \tag{6}$$

These transverse electromagnetic solutions are well known by the names of ordinary mode (O-mode) and extraordinary mode (X mode) [8]. The first mode does not have any resonance and propagate for $\omega > \omega_{pe}$ because of the cut-off and the second one has two cut-offs and two resonances. According to the phase velocity ω/k, it decomposes in fast (F) and slow (S) as shown in Fig 3 and in Fig 4.

The two branches of propagation (ordinary and extraordinary) appear and we can see that the ordinary mode propagates for frequencies such that $\omega > \omega_{pe}$. The extraordinary mode is propagated for $\omega_L < \omega < \omega_{uh}$, evanescent for $\omega_{uh} < \omega < \omega_R$. It becomes propagative when $\omega > \omega_R$. With ω_R, ω_L are the cutoff frequencies of the X mode, called right and left modes, defined by:

$$\omega_{R,L} = \frac{1}{2}\left[\mp\omega_c + \left(\omega_c^2 + 4\omega_p^2\right)^{1/2}\right] \tag{7}$$

The X mode has a cold resonance ($N_\perp \to \infty$), given by:

$$\omega_{uh} = \sqrt{\omega_c^2 + \omega_p^2} \tag{8}$$

This resonance is called upper hybrid (UH) is not available if $\omega > \omega_c$. There is also a lower hybrid resonance [9], it is well below the electron cyclotron frequency domain and therefore not interfere here.
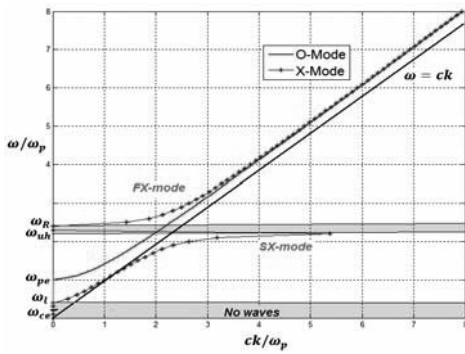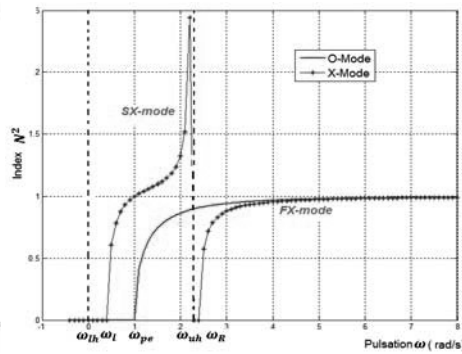

Fig. 3. the dispersion diagram

Fig. 4. $N^2 = f(\omega)$ for perpendicular propagation

3. Energy Transfer

The mechanism of wave energy transfer by absorption is schematized on the Fig.5. In a tokamak, the production of an electromagnetic power is usually made by gyrotrons for ECRH and transported to the plasma by transmission lines. What causes an excitation of a plasma wave at the edge. This wave traveling toward the center by carrying the power and in a resonance layer near of $\omega = \omega_c$, it will be absorbed by transferring its energy to the resonant electrons which in turn collide with other plasma electrons. Finally these particles thermalize, thus increasing the temperature of the bulk plasma (see Fig.5).
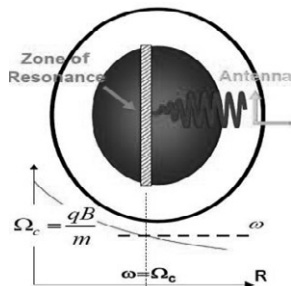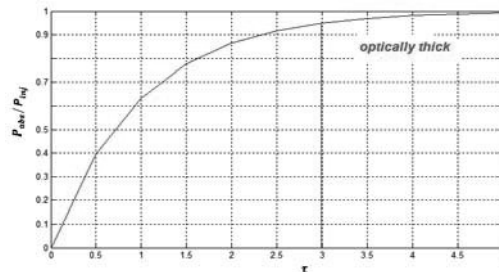

Fig.5. Principle of ECR heating

Fig.6. Fraction of power absorbed as a function of τ, ($\tau > 3$) [2].

3.1. Absorption of Electron Cyclotron Wave in Plasma

In fact, the cyclotron resonance does not appear explicitly in the cold model. Because the cyclotron resonance is, in its principle, an interaction between the wave and particle motion. In other words, it involves the microscopic structure of the plasma. We shall use the kinetic theory, to accurately reflect the phenomena occurring at the particle scale. The hot plasma model under certain approximations, leads to a new expression of dielectric tensor can be expressed by a correction of the type:

$$\overline{\overline{K}}_{hot} = \overline{\overline{K}}_{cold}\big(\omega, B_0, n_{e,0}\big) + \widetilde{K}\big(\omega, B_0, n_{e,0}, T_{e,0}\big) \tag{9}$$

The hot correction $\widetilde{K}$ depends explicitly on the wave vector $\vec{k}$ and the electron temperature at equilibrium, $T_{e,0}$. To calculate the elements of $\overline{\overline{K}}_{hot}$, we start from the relativistic Vlasov equation [2], [10]. In the relativistic formalism, the distribution function of electrons is written as $f_e(\vec{r}, \vec{p}, t)$ with the relation $\vec{p} = m_{e,0}.\gamma.\vec{v}$ where $m_{e,0}$ is rest mass. The distribution function is solution of the relativistic Vlasov equation given by:

$$\frac{\partial f_e}{\partial t} + \frac{\vec{p}}{m_e\gamma}\frac{\partial f_e}{\partial \vec{r}} - e\left(\vec{E} + \frac{1}{m_e\gamma}\vec{p}\wedge\vec{B}\right)\frac{\partial f_e}{\partial \vec{p}} = 0 \qquad (10)$$

Where $m_e^2 = m_{e,0}^2 + (p/c)^2 = m_{e,0}^2\gamma^2$ is the relativistic mass of the electron.

3.2. Relativistic Dielectric Tensor

The distribution function f_e is written as $f_e(\vec{r}, \vec{p}, t) = f_{e,0}(\vec{p}) + f_{e,1}(\vec{r}, \vec{p}, t)$ the sum of two distribution functions f_{e0} for equilibrium state and f_{e1} for the perturbed state. Similarly to distribution function f_e , the magnetic and electric fields [6], can be written as $\vec{B} = \overrightarrow{B_0} + \overrightarrow{B_1}$ and $\vec{E} = 0 + \overrightarrow{E_1}$. A perturbed state of linearized Vlasov equation takes the form

$$\frac{df_{e,1}}{dt} = \frac{\partial f_{e,1}}{\partial t} + \frac{\vec{p}}{m_e}\frac{\partial f_{e,1}}{\partial \vec{r}} + \frac{e}{m_e}\left(\vec{p}\wedge\overrightarrow{B_0}\right)\frac{\partial f_{e,1}}{\partial \vec{p}} = -e\left(\vec{E} + \frac{\vec{p}\wedge\vec{B_1}}{m_e}\right).\frac{\partial f_{e,0}}{\partial \vec{p}} \qquad (11)$$

The integration of equation (11) gives the relativistic dielectric tensor:

$$K_{ij} = \delta_{ij} - \frac{\omega_p^2}{\omega^2}\frac{\mu^2}{2k_2(\mu)}\int_{-\infty}^{+\infty}d\overline{p}_{II}\int_0^{+\infty}\overline{p}_\perp d\overline{p}_\perp \frac{e^{-\mu\gamma}}{\gamma}\sum_{n=-\infty}^{n=\infty}\frac{P_{i,j}^n(p_\perp,p_{II})}{\gamma - n\frac{\omega_{ce}}{\omega} - n_{II}\overline{p}_{II}} \qquad (12)$$

Where $\overline{p} = p/(m_{e,0}c) = \overline{p}_\perp + \overline{p}_{II}$, $n_{II} = ck_{II}/\omega$ is the index refraction for parallel direction to $\overrightarrow{B_0}$ and $k_n(z)$ is the modified Bessel function of second kind (or McDonald function) of index n (here $n = 2$) and argument z.

If we decompose the dielectric tensor in hermitian and anti-hermitian parts respectively as $\overline{\overline{K}} = \overline{\overline{K}}_h + i\overline{\overline{K}}_a$. And if one decompose the hot correction $\widetilde{K}$ in real and imaginary part as $\widetilde{K} = \widetilde{K}' + i\widetilde{K}''$. The expression (9) can be written:

$$\overline{\overline{K}}_{hot} = \underbrace{\begin{pmatrix} S + \widetilde{K}_q' & -i\left(D - \widetilde{K}_q'\right) \\ i\left(D - \widetilde{K}_q'\right) & S + \widetilde{K}_q' \end{pmatrix}}_{hermitian} + i\underbrace{\begin{pmatrix} \widetilde{K}_q'' & i\widetilde{K}_q'' \\ -i\widetilde{K}_q'' & \widetilde{K}_q'' \end{pmatrix}}_{anti-hermitian} \qquad (13)$$

It can be shown that the first hermitian part $\overline{\overline{K}}_h$ characterizes the propagation while the second anti-hermitian part $\overline{\overline{K}}_a$ characterizes the absorption [11]. If $T_e \to 0$, we obtain $\overline{\overline{K}}_a = 0$ and $\overline{\overline{K}}_h = \overline{\overline{K}}_{cold}$; which justifies the use of the cold approximation to describe wave propagation [11].

3.3. Absorption Coefficient

We take the viewpoint of geometrical optics by considering a plane monochromatic wave of type $\vec{E}(\vec{r}, t) = \vec{E}(\vec{k}, \omega)\,exp\{i[\vec{k}.\vec{r} - \omega t]\}$ for which one trying to describe the dissipation by introducing the concept of absorption coefficient. For there to be absorption, it is necessary that the wave vector $\vec{k}$ is complex like $\vec{k} = \overrightarrow{k'} + i\overrightarrow{k_a''}$ and its imaginary part is nonzero, $\overrightarrow{k_a''} = (\omega/c)\overrightarrow{N''} \neq 0$. Then the absorption coefficient [8] is given by

$$\alpha = -2k_a^{''} \cdot \frac{\vec{v}_g}{v_g} \qquad (14)$$

With $\overrightarrow{\boldsymbol{v}_g} = \frac{d\vec{r}}{dt}$ is the group velocity. For the explicit calculation of the absorption coefficient, we introduce another approach based on energy conservation, using the anti-hermitian part of the dielectric tensor. Poynting's theorem [12] writes:

$$\frac{\partial W_{0,t}}{\partial t} + \vec{\nabla}.\vec{S}_{0,t} = \frac{\partial}{\partial t}\frac{1}{2}\left(\frac{|\overrightarrow{B_t}|^2}{0} + \varepsilon_0|\vec{E}_t|^2\right) + \frac{1}{\mu_0}\vec{\nabla}.Re\left(\vec{E}_t \wedge \vec{B}_t\right) = -\vec{j}_t.\vec{E}_t \qquad (15)$$

Where $\partial W_{0,t}/\partial t$, the instantaneous energy density contains the magnetic $|\vec{B}_t|^2/(2\mu_0)$ and electrostatic $\frac{1}{2}\varepsilon_0|\vec{E}_t|^2$ energies respectively. $\vec{S}_{0,t}$ is the instantaneous Poynting vector in vacuum describing the flow of electromagnetic energy. The source term, $-\vec{j}_t.\vec{E}_t$, describes the interactions of the wave with the plasma. By performing the time average over a few periods of oscillations $\langle\vec{E}_t\rangle t = E_1(\vec{r})\,exp(i\vec{k}.\vec{r})$, and separating explicitly the hermitian and anti-hermitian parts of dielectric tensor introduced into the source term, we can be extracted from equation (15) the absorption coefficient as:

$$\alpha = \frac{\varepsilon_0\omega\overrightarrow{E_1^*}\overleftrightarrow{K_a}\overrightarrow{E_1}}{|\vec{S}|} \qquad (16)$$

Where $\overrightarrow{E_1^*}$ is the complex conjugate of $\overrightarrow{E_1}$ and $\vec{S} = \vec{S}_0 + \vec{Q}_s$ with $\vec{S}_0 = \frac{1}{4\mu_0}Re\left(\overrightarrow{E_1^*} \wedge \overrightarrow{B_1} + \overrightarrow{E_1} \wedge \overrightarrow{B_1^*}\right)$ and $\vec{Q}_s = -\frac{1}{4}\varepsilon_0\omega\overrightarrow{E_1^*}\frac{\partial\overrightarrow{K_h}}{\partial k}.\overrightarrow{E_1}$.

Optical depth or optical thickness is a measure of transparency and is defined as the integral of the absorption coefficient α along the trajectory s of the wave like $\tau = \int -\alpha \qquad$, [2], [3], [9]. The total absorbed power P_{abs} in the plasma can then be written as

$$P_{abs} = P_{inj}\left(1 - exp(-\ )\right) \qquad (17)$$

We can see an illustration of the function P_{abs}/P_{inj} in Figure 6 where we define that the plasma is optically thick when $\tau > 3$, that is to say the fraction of absorbed power $P_{abs}/P_{inj} > 95\%$.

The relation of resonance is given by the relativistic cyclotron resonance condition of energy exchange between the wave electron cyclotron and plasma as follows:

$$\gamma - k_{II}v_{II} - n\frac{\omega_{ce}}{\omega} = 0 \qquad (18)$$

The term $k_{II}v_{II}$ describes longitudinal Doppler shift [5]. The term $n\omega_{ce}/\omega$ describes the gyration of the electron; n is the order of the harmonic excited. This relation expresses the equality between the frequency of the wave and the relativistic cyclotron frequency of rotation corrected by the Doppler shift which caused by the electron parallel velocity. The energy of resonant electrons at ω_{ce} and given n_{II}can be written as:

$$E = m_e c^2 (k_{II}v_{II} + n\frac{\omega_{ce}}{\omega} - 1) \qquad (19)$$

3.4. Curve of resonance:

In the relativistic case, the curves of resonance between electron cyclotron waves and plasma are semi-ellipses as shown in Fig.7. (a) in the momentum space $(\bar{p}_{II}, \bar{p}_\perp)$ with the equation derived from (18) is written [13] as

$$\frac{(\bar{p}_{II}-\bar{p}_{II,0})^2}{\alpha_{II}^2} + \frac{\bar{p}_\perp^2}{\alpha_\perp^2} = 1 \qquad (20)$$

With $\bar{p}_{II,0} = \frac{N_{II}(n\omega_{ce}/\omega)}{1-N_{II}^2}$ for the center of ellipse and the lengths of its semi-axis are given by

$$\alpha_{II} = \frac{\sqrt{(n\omega_{ce}/\omega)^2 - (1 - N_{II}^2)}}{1 - N_{II}^2}, \, \alpha_{\perp} = \frac{\sqrt{(n\omega_{ce}/\omega)^2 - (1 - N_{II}^2)}}{\sqrt{1 - N_{II}^2}} \tag{21}$$

- If $(n\omega_{ce}/\omega)^2 < (1 - N_{II}^2)$ any exchange of energy between the wave and the plasma is prohibited.
- If $(1 - N_{II}^2) < (n\omega_{ce}/\omega)^2 < 1$, the wave may transfer its energy to plasma. The absorption is then traditionally described as "up-Shifted".
- If $(n\omega_{ce}/\omega)^2 > 1$, we can expect that the absorption takes place mainly in the vicinity of $\bar{p}_{II} = \bar{p}_{II,0} - \alpha_{II}$. Under these conditions, the absorption is described as "down-shifted".

The Fig.7 (b) is the same as Fig.7.(b) but for perpendicular propagation, so the equation (18) becomes:
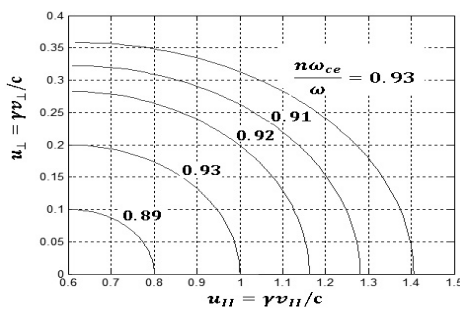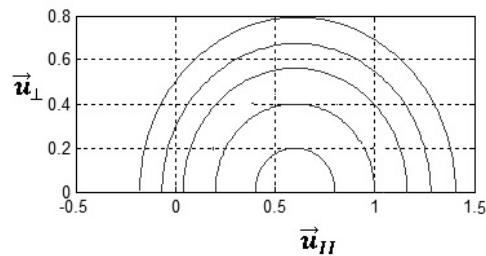
$$\gamma - n\frac{\omega_{ce}}{\omega} = 0 \tag{22}$$

Fig.7. Resonance curves ($N_{II} = 0.5$) (a) for oblique propagation;   (b) for perpendicular propagation

5. Summary

The transfer of energy to the plasma is made by the wave interaction with cyclotron moving electrons in resonance. This transfer of energy by absorption appears as kinetic energy of the electrons which increases the thermal motion in the plasma and hence plasma heating. The application of EC waves to plasmas rests on a wide base of theoretical work which progressed from simple cold plasma models to hot plasma models with fully relativistic physics to quasilinear kinetic Vlasov models. This technique is used in tokamak machines as ITER for additional heating (ECRH) and the generation of non-inductive current drive (ECCD). The power absorbed depends on the optical depth which in turn depends on coefficient of absorption and the order of the excited harmonic for chosen mode generally in perpendicular propagation to magnetic field. The relation of resonance can determine if the curve of resonance is ellipse or a circle according to the propagation oblique or perpendicular.

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