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Mathematical Model for Magnetohydrodynamic Equilibrium Study of Tokamak Plasma

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Abstract – *This paper presents a study of the interaction of a magnetic field with plasma in tokamak. Our aim is to determine the equilibrium state; in other words the states where it can be a good confinement by using the magnetohydrodynamic (MHD) approach.*

***Keywords**: MHD, Tokamak, Magnetic confinement, Free borders*

I. Introduction

Tokomak is a machine designed to carry out controlled thermonuclear fusion, this reaction requires a good confinement, for that one uses what is called the MHD which makes it possible to give an idea on the geometry of plasma.

In our study, one finds that the simplification of the equations of the ideal MHD leads to the equation of Grad Shafranov, his resolution is done in an orthogonal space of flows, where it is reduced to a Laplacian.

These simplifications enable us to propose a mathematical model which describes the plasma equilibrium of tokamak. For the numerical resolution of this problem, one uses the finite element method.

II. Interaction of a Conductng Medium with a Magnetic Field

The evolution of the magnetic field in a conducting medium, supposed here plasma is represented by the following equation (1):

$$\frac{\partial \vec{B}}{\partial t} = r\vec{o}t\left(\vec{V} \wedge \vec{B}\right) + \eta \Delta \vec{B} \quad (1)$$

where:

$\vec{B}$: Magnetic field (Tesla);

$\vec{V}$: Propagation velocity. (m.s^{-1}) ;

$\eta = \left(\mu_0 \sigma\right)^{-1}$: Magnetic coefficient of diffusion (m.s^{1}).

It is obtained starting from the equation of Maxwell-Ampere and the equation of induction [1].

The two terms which appear in the equation (1) describe two different mechanisms: convection and diffusion.

Let us suppose now that the second term of the equation (1) is null, this borderline case is obtained by considering that electric conductivity σ is infinite.

I.e. plasma is perfectly conductive (case of tokamak). In this case one places oneself within the framework of the magnetohydrodynamic (ideal MHD), where the field moves by convection (Fig. 2), which is translated by the equation (2):

$$\frac{\partial \vec{B}}{\partial t} = r\vec{o}t\left(\vec{V} \wedge \vec{B}\right) \quad (2)$$

Convection by implicit centered: Δt=0.33333; Δx=0.05

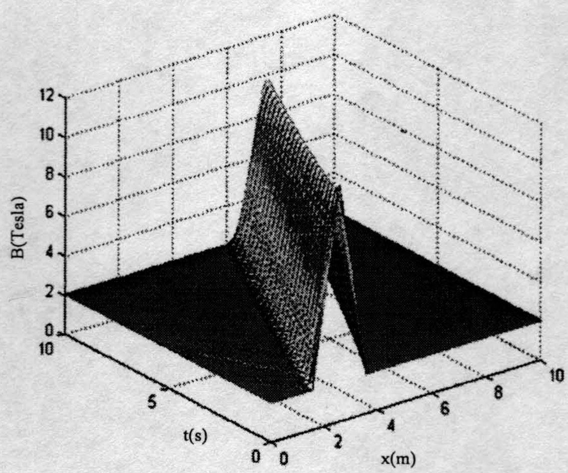

Fig. 1. Evolution of magnetic field by convection

Magnetohydrodynamic (MHD) is a phenomenon which is used to move the molecules and the atoms ionised by the action of the forces of Laplace (Magnetic and electric), where the equations of the MHD are obtained by the linearisation of the equation of the hydrodynamics and the Maxwell's equations [3].

Equations of the ideal MHD (no resistive), where plasma is infinitely conductive and perfectly compressible is given by:

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$$\begin{cases} div(\vec{B}) = 0 \\ r\vec{o}t(\vec{B}) = \mu_0 \vec{j} \\ \frac{\partial \vec{B}}{\partial t} = r\vec{o}t(\vec{V} \wedge \vec{B}) \\ \vec{j} \wedge \vec{B} = gr\vec{a}d(p) \end{cases} \quad (3)$$

III. Application

In tokamak plasma, the lines of the magnetic field must be closed on them same to have a good confinement, for that one considers that plasma is surrounded by a certain thickness of vacuum which is surrounded by a conducting wall, and then one applies a strong magnetic field. One distinguishes two groups from equations: those which describe plasma are the equations of the MHD and those which describe the vacuum are the Maxwell's equations.

a. Magnetic confinement of plasma

Confinement in plasma of tokamak is primarily controlled by the diffusive losses of particles, where the magnetic field acts on a particle via the force of Lorentz (7):

$$\vec{F} = q\vec{v} \wedge \vec{B} \quad (4)$$

If the components of the vector speed are perpendicular to the field that gives place to a force, such as the particle is accelerated with a constant acceleration, where it will carry out a circular motion, in this case the particle will have a perfect confinement.

However in the direction parallel with the field, the particle does not undergo any force, it will not be confined. The total movement will be the sum of a rectilinear motion without acceleration superimposed on a circular motion which gives rise to a helical movement. In other words the particle describes a propeller around the line of the magnetic field, one is then in a configuration where the direction of the magnetic field is purely toroïdal (Fig. 2).

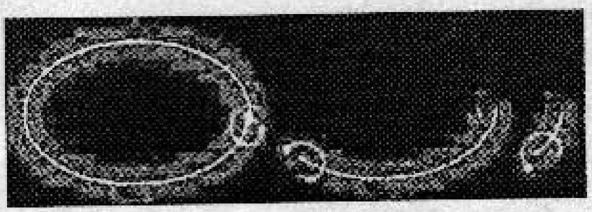

Fig. 2. Trajectory particle in a magnetic field

b. Equilibrium inside plasma

To be able to control the confinement of plasma in tokamak one uses the equations of the MHD. The resolution of this system of equations in plasma is associated boundary conditions adequate which leads to the equation of Grad Shafranov.

To be able to simplify the equation of Grad Shafranov, one uses the principle of orthogonalisation where the lines of flow are encased contours one in another, then it seems possible to use them like a grid.

This family of contours is axisymmetric, and then it has just determined a flux space.

The equation of grad Shafranov in the space of flow is given by:

$$\Delta\left(\frac{\psi}{R}\right) = -\left(\mu R \frac{dp}{d\psi} + \frac{F}{R}\frac{dF}{d\psi}\right) \quad (5)$$

The resolution in flux space is equal to solve a Laplace equation, in other words the equation of Grad Shafranov is reduced to a Laplacian.

$$\nabla^2 U = 0 \quad (6)$$

IV. Formulation of the Problem

In order to determine the behaviour of flux of the magnetic field in plasma, we propose a mathematical model which describes the confinement of plasma in tokamak:

$$\begin{cases} -\Delta u = j & dans\ w \\ -\Delta u = 0 & \Omega / w \\ u = 0 & \partial\Omega \end{cases} \quad (7)$$

where:

j: the current density

u: flux of the magnetic field

w : Area occupied by plasma, Ω total field and $\partial\Omega$ the boundary.

This model contains equations of the type EDP, then for the resolution of this system of equation, one uses the finite element method.

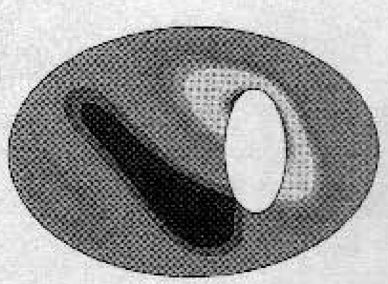

Fig. 3. Confinement of plasma by a no uniform magnetic fiel

In the Fig. 2 one observes that the intensity of the magnetic field is not uniform, where plasma moves towards the area where the intensity is low, on the other hand in the Fig. 4 where the intensity of the magnetic field is uniform, plasma is in the center and will have the shape of a torus.

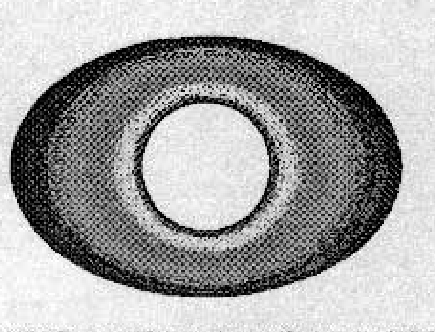

Fig. 4. Confinement of plasma by a uniform magnetic field

Let us suppose that plasma has the geometry of a disc, where, $j=\lambda$ is a parameter ($\in\Re$), $\Omega=B(0,1)\subset\Re^2$ is a disc.

If Ω has the geometry of a disc, the mathematical model is reduced as:

$$\begin{cases} u''+\frac{1}{r}u'=-\lambda & if\ 0\prec r\prec r_0 \\ u''+\frac{1}{r}u'=0 & r_0\prec r\prec 1 \\ u(1)=0,\ u'(0)=0,\ u(r_0)=1 & \end{cases} \quad (8)$$

Seeking radial solutions, i.e. $u=u(r)$ where r the ray of disc.

$$u(r)=\begin{cases} 1-\frac{\lambda}{2}r_0^2\, ln\left(\frac{r}{r_0}\right) & \text{In the vacuum} \\ 1+\frac{\lambda}{4}\left(r_0^2-r^2\right) & \text{In the plasma} \end{cases} \quad (9)$$

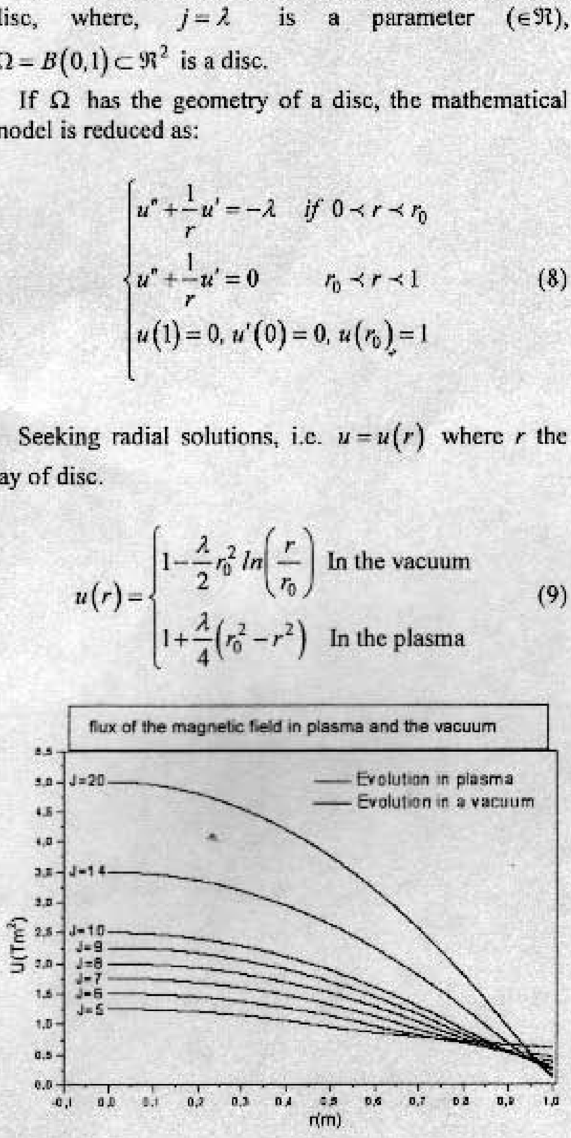

Fig. 5. Behavior of flux of magnetic field in plasma and the vacuum

Such as one can trace these solutions to see the behavior of flux of the magnetic field in plasma and the vacuum.

One observes in the Fig. 5 for different value from j, one will have a set of encased contours one in another, one concludes that these contours represent magnetic surfaces confining plasma in a toroidal structure.

V. Conclusion

From this study, one concludes that the MHD is a very powerful tool when it is a question of dealing with the problems of balance in plasma of tokamak, such as having balance it is necessary to have a good containment, for that it is necessary to apply a strong magnetic field.

The modelling of the equations of the MHD makes it possible to translate the containment of plasma by a problem at free border.

To have a good confinement, it is necessary that the magnetic field is uniform, and plasma must have the configuration of a disc.

If plasma has the geometry of a disc, the density flux of magnetic field is maximum within plasma and weak in the vacuum, such as the value of flow is always constant $u(r_0)=1$ the border of plasma.

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