Contribution to the Study of Optical Properties of a Dielectric Medium (Atomic Vapor) Using the Lorentz Model

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Abstract
Optical properties of a dielectric medium consisting of an atomic vapor are investigated theoretically using the model of elastically bound electrons. This model describes the interaction of an electromagnetic field with the bound electrons to the vapor atoms [7]. In this paper, we propose a formalism which takes into accurate the effect of the number of electrons on the vapor index. We use the approximation of free electrons (no interaction between free electrons).
Keywords*: atomic vapor, electromagnetic field, Maxwell's equation, elastically bound electron, refraction index.*

1. Introduction

The atomic vapor is considered as not very dense dielectric medium comprising N atoms per unit of volume [1]. By supposing that the atom has several electrons of the same mass m , of the same load and of the same own pulsation (thus, for example, all the outer-shell electrons of an atom can have the same behavior) [2]. These loads are likely to move under the action of the electric field of an electromagnetic wave. In our study, one is interested has to determine the optical properties of the medium while being based on the Lorentz model [5] and to study the influence of the electrons number on the index of the vapor.

2. Détermination of the atomic vapor index

1) Model of the elastically bound electron:
Within the framework of this model, the electron is subjected to:
An elastic force of recall, proportional to its displacement $\vec{r}$ compared to its position of balance:

$$\vec{f} = -m\omega_0^2\vec{r}. \tag{1}$$

Where
$\vec{r}$: radius vector of the electron.
ω_0 : own pulsation of the electron.
m : mass of the electron.
A force intended to account for the dissipative phenomena of energy:

$$\vec{f} = -m\gamma\vec{v}. \tag{2}$$

A force of LORENTZ created by the electromagnetic field of the wave, where neglecting us, for the non relativistic electron, the influence of the magnetic term [6]:

$$\vec{f} = -e\vec{\mathrm{E}}. \tag{3}$$

This model describes the movement of the electron which east governs by a differential equation of the form:

$$\left(\omega_0^2 - \omega^2\right)\vec{r} + j\gamma\omega\vec{r} = -\frac{e}{m}\vec{\mathrm{E}}. \tag{4}$$

The solution of this equation presents the displacement of the electron:

$$\vec{r} = \frac{-e/m}{\omega_0^2 - \omega^2 + j\gamma\omega}\vec{\mathrm{E}}. \tag{5}$$

In the vapor no charged and no conducting, the Maxwell's equations are written [3]:

$$\begin{cases} div\vec{D} = 0 & rot\vec{\mathrm{E}} = \dfrac{-\partial\vec{B}}{\partial t} \\ div\vec{B} = 0 & rot\vec{H} = \dfrac{\partial\vec{D}}{\partial t} \end{cases}. \tag{6}$$

The equation of propagation of the electric field in the medium is given by:

$$\Delta\vec{\mathrm{E}} - \varepsilon_0\mu_0\frac{\partial^2\vec{\mathrm{E}}}{\partial t^2} = \mu_0\frac{\partial^2\vec{p}}{\partial t^2} - \frac{1}{\varepsilon_0}gr\vec{a}d\,div\vec{P}. \tag{7}$$

The electron forms with the core a dipole of dipole moment $\vec{p} = -e\vec{r}$ and consequently a polarization of the medium:

$$\vec{P} = \frac{\varepsilon_0\Omega^2}{\omega_0^2 - \omega^2 + j\gamma\omega}. \tag{8}$$

Where $\Omega^2 = \dfrac{Ne^2}{m\varepsilon_0}$

Of other by, one has

$$\begin{cases} \vec{P} = \varepsilon_0 \chi \vec{\mathrm{E}} \\ D = \varepsilon_0 \vec{\mathrm{E}} + \vec{P} \end{cases}. \quad (9)$$

One finds:

$$\varepsilon_r = 1 + \chi = 1 + \frac{\omega^2}{\omega_0^2 - \omega^2 + j\gamma\omega}. \quad (10)$$

For monochromatic progressive plane wave studied, the electric field is written:

$$\vec{E} = \vec{E}_0 \exp j\left(\omega t - \vec{k}\vec{r}\right). \quad (11)$$

with:

$$\begin{cases} \dfrac{\partial}{\partial t} = j\omega \\ \vec{\nabla} = -jk\vec{z} \end{cases}. \quad (12)$$

The dispersion relation is obtained:

$$k^2 = \frac{\omega^2}{c^2}\varepsilon_r. \quad (13)$$

With $k_0 = \dfrac{\omega}{c}$, is the module of the wave vector for propagation in the vacuum, one obtains:

$$k^2 = k_0^2 \varepsilon_r. \quad (14)$$

This makes it possible to define the medium index:

$$\left(\frac{k}{k_0}\right)^2 = n^2 = \varepsilon_r. \quad (15)$$

It comes that:

$$n^2(\omega) = \left[1 + \frac{\Omega^2}{\omega_0^2 - \omega^2 + j\gamma\omega}\right]. \quad (16)$$

The index of the medium is a complex number from where $n = n_1(\omega) - jn_2(\omega)$, the electric field of the wave being propagated towards Z positive of the axis is written: $\vec{E} = \vec{E}_0 \exp\left(-\dfrac{\omega}{c}n_2 z\right)\exp j\omega\left(t - \dfrac{n_1 z}{c}\right).$

The real part n_1 is the refraction index of the medium for the pulsation ω, and the imaginary part n_2 which characterizes the absorption of the electromagnetic wave by the medium is named extinction index.

In rise with the square in n, and by neglecting $n_2^2(\omega)$ in front of $n_1^2(\omega)$, one obtains by identification:

$$\begin{cases} n_1^2(\omega) = 1 + \Omega^2 \dfrac{\left(\omega_0^2 - \omega^2\right)}{\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\omega^2} \\ n_1(\omega)n_2(\omega) = \dfrac{\Omega^2}{2}\dfrac{\gamma\omega}{\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\omega^2} \end{cases}. \quad (17)$$

The plotted curves so below indicate the refraction and extinction indexes variation of the vapor according to the pulsation omega.

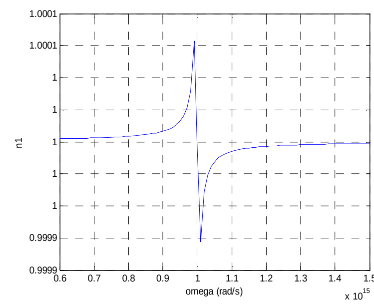

Fig.1: Real index according to the wave pulsation.

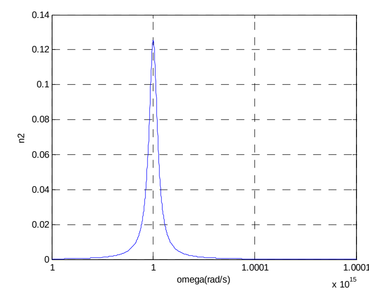

Fig.2: Imaginary Index according to the wave pulsation.

One can check that the real part of the index is strictly null with resonance, maximum or minimal approximately with a width of resonance. Such a curve is called dispersion curve; it describes in a rather realistic way the refraction index variation in the vicinity of resonance.

The imaginary part is on the other hand maximum for $\omega = \omega_0$, it decrease quickly as soon as ω deviates from ω_0. This curve is called Lorentziène: it is symmetrical compared to the axis passing by its maximum located at $\omega = \omega_0$. It is the curve of absorption.

3. Study of the index according to the number of electrons

1) Imaginary index (extinction index)

Generally, a medium contains several electrons which are likely to move under the action of the electric field of an electromagnetic wave [4].

By making $n_1(\omega)=1$ in the product $n_1(\omega)n_2(\omega)$ in equation (17), the extinction index of the medium is thus written:

$$n_2(\omega)=\frac{N_e\Omega^2}{2}\frac{\gamma\omega}{\left(\omega_0^2-\omega^2\right)^2+\gamma^2\omega^2} \qquad (18)$$

N_e is the electrons number released by the atom.
In our study, one bases oneself on an electrons number which varies from one up to four. While replacing respectively N_e by his values, one obtains four relations which represent the variation of the index n_2 according to ω for the different ones goes theirs of N_e.

$$\begin{cases} n_2(\omega)=\dfrac{\Omega^2}{2}\dfrac{\gamma\omega}{\left(\omega_0^2-\omega^2\right)^2+\gamma^2\omega^2} \\ n_2(\omega)=\Omega\dfrac{\gamma\omega}{\left(\omega_0^2-\omega^2\right)^2+\gamma^2\omega^2} \\ n_2(\omega)=\dfrac{3\Omega^2}{2}\dfrac{\gamma\omega}{\left(\omega_0^2-\omega^2\right)^2+\gamma^2\omega^2} \\ n_2(\omega)=2\Omega^2\dfrac{\gamma\omega}{\left(\omega_0^2-\omega^2\right)^2+\gamma^2\omega^2} \end{cases}. \qquad (19)$$

The figure so below gives the n_2 variation according to the pulsation. It is noticed directly that the absorption increases according to the electrons number.
The detailed examination of the curve shows that the difference between the peaks obtained of the adjacent curves is not constant, but increases in a linear way with the number of electrons (figure 4).
The linear variation enters Δn_2 and the electrons number translates the fact that n_2 increases in a nonlinear way with N_e what is illustrated on figure (5):

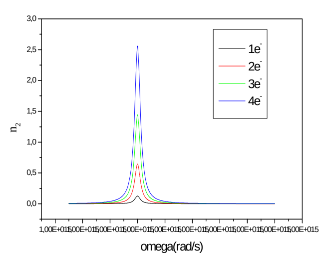

Fig.3: Extinction index according to the pulsation.

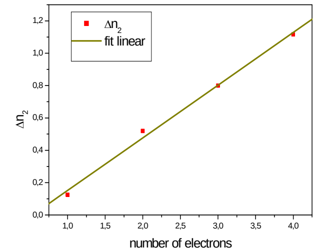

Fig.4: Δn_2 according to the electron number.

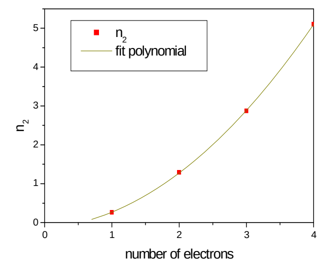

Fig.5: Variation of the extinction index according to the electrons number.

From this last curve, one can deduce a behavior law of this nonlinear variation, it is clear that it follows a polynomial law of order 2 of the type:

$$y=a+b_1x+b_2x^2. \qquad (20)$$

Thus, one can rewrite it according to:

$$n_2=0.032+0.0623N_e+0.149N_e^2. \qquad (21)$$

From this law, one can evaluate the extinction index of the not very dense medium which contains an electrons number higher than 1.
2) Real Index (refraction index)
The refraction index is given by:

$$n_1^2(\omega)=1+\frac{N_e\Omega^2\left(\omega_0^2-\omega^2\right)}{\left(\omega_0^2-\omega^2\right)+(\gamma\omega)^2}. \qquad (22)$$

In the same way, while replacing respectively N_e by his values, one obtains four equations, whose their layouts are given on figure (6):

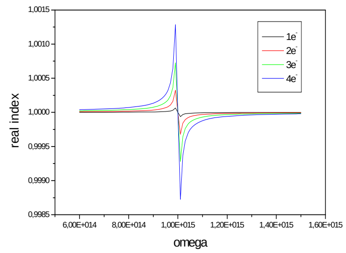

Fig.6: refraction index according to the pulsation of the wave.

One notes according to this figure that the function $n_1(\omega)$ believes more and more according to the electron number, (one considers only the left branch of the figure for the results interpretation; where one speaks about normal dispersion).

The dispersion is thus significant when the studied medium comprises several electrons.

The difference Δn_1 varies in the same way that Δn_2 ; it increases linearly with the electrons number (figure 7), which means that the refraction index increases not linearly with N_e (figure8):

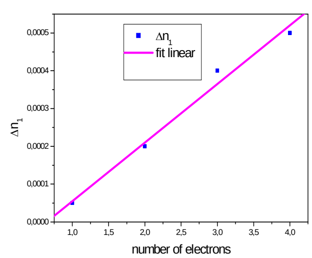

Fig.7: Δn_1 according to the electrons number.

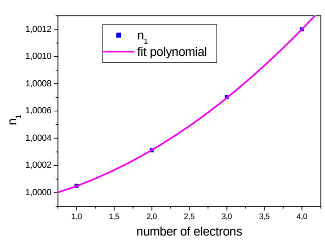

Fig.8: Evolution of the refraction index according to the electrons number.

One can then define a second behavior law which represents the nonlinear variation between the refraction index and the electrons number. It is written in the following form:

$$n_1 = 0.999 + 8.4\times10^{-5} N_e + 6\times10^{-5} N_e^{\ 2}. \quad (23)$$

4. Conclusion.

One can conclude that the model suggested enabled us to define the optical properties of the atomic vapor (dielectric medium which is subjected to the action of an incident electromagnetic wave) and to study of this fact the influence of the electrons number on these optical properties such as the refraction index.

Thus, the objectives of the laws found through this study is the possibility of determining the optics properties to know the refraction index, susceptibility and the permittivity of the atomic vapor for each quantity of electrons.

The variation of the index with the electrons number is a property which can be exploited as selection criterion of the atomic vapor which is often used in the experimental devices intended for the development of materials such as the thin layers and the optical characterization (determination of the experimental index of the index of refraction).

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