Praise Worthy Prize

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Transfers of Electromagnetic Energy in Homogeneous Plasma

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Abstract – *Transfers of energy during the propagation of an electromagnetic wave in homogeneous and stationary plasma are described by the instantaneous Poynting theorem, using Maxwell equations. A plasma being a complex middle to describe; thus it will be necessary to work on meanly quantities. The energy of the wave carried by the movement of particles is represented by the kinetic flux.*

Keywords: *Transfer, Energy, Electromagnetic, Plasma, Poynting*

Nomenclature

$\vec{D}$	The electric displacement
$\vec{H}$	The magnetic induction
$\vec{B}$	The magnetic field
$\vec{E}$	The electric field
ρ_{ext}	The density of charge
$\vec{j}$	The density of current (in this article we call only current)
ε_0	The dielectric permittivity in the emptiness
μ_0	The magnetic permeability in the emptiness
c	The speed of light in the emptiness
k	The vector of wave
$\overline{\overline{\varepsilon}}$	The dielectric tensor
$\overline{\overline{\sigma}}$	The tensor of conductivity of plasma
$\overline{\overline{I}}$	The tensor identity
ω	The pulsation of wave
ω_{pe}	The electronic plasma pulsation
ω_{ce}	The electronic cyclotron
N	The refractive index
W	The electromagnetic energy density
T	The kinetic flux
$\vec{S}_0$	The vector of Poynting
P_{abs}	The absorbed power

I. Introduction

Plasma is susceptible to propagate a big number of oscillation's mode. This is due to the Colombian interaction between the charged particles that provokes strength of recall as soon as plasma departs the electric neutrality. The thermal movement of particles modifies the dielectric constant of plasma, and especially gives back possible exchanges of energy [1] brings in wave and particle that can not be analyses that with the kinetic theory.

II. The Maxwell's Equations

The electric and magnetic fields characterizing an electromagnetic wave [2]-[3] are the shape:

$$\begin{cases} \vec{E} = \overline{E_0}\, exp\, j\left(\omega t - \vec{k}\vec{r}\right) \\ \vec{B} = \overline{B_0}\, exp\, j\left(\omega t - \vec{k}\vec{r}\right) \end{cases} \tag{1}$$

To the breast of plasma, it is described by the Maxwell's equations that we write under the shape:

$$\vec{\nabla}.\vec{D} = \rho_{ext} \tag{2}$$

$$\vec{\nabla}.\vec{B} = 0 \tag{3}$$

$$\vec{\nabla}\Lambda\vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{4}$$

$$\vec{\nabla}\Lambda\vec{H} = \vec{j}_{ext} + \frac{\partial \vec{D}}{\partial t} \tag{5}$$

Using the relations:

$$\vec{j}_{tot} = \vec{j} + \vec{j}_{ext} \tag{6}$$

$\vec{j}_{ext}$ the current coming from exterior sources.

Where $\vec{j}$ is the current that settles in plasma in presence of the electromagnetic excitation and:

$$\vec{j}_{tot} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \vec{j}_{ext} + \frac{\partial \vec{D}}{\partial t} \tag{7}$$

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We find:

$$\frac{\partial \bar{D}}{\partial t} = \bar{j} + \varepsilon_0 \frac{\partial \bar{E}}{\partial t} \quad (8)$$

Equation of Propagation and Relation of Scattering

The equation of propagation of an electromagnetic wave in plasma ensues of Maxwell's equation and it is express by the relation:

$$\vec{\nabla}\Lambda\vec{\nabla}\Lambda\bar{E} + \mu_0 \frac{\partial}{\partial t}\bar{j} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\bar{E} = \vec{0} \quad (9)$$

As you known $\bar{E}$ and $\bar{j}$ are respectively the electric field of the wave and the current in the plasma due to the displacement of charged particles under the action of the wave. In the case of a plane monochromatic wave of the shape $e^{i(\bar{k}.\bar{r}-\omega.t)}$, the (9) is written under the shape:

$$\bar{k}\Lambda\bar{k}\Lambda\bar{E} + \frac{\omega^2}{c^2}\bar{\bar{\varepsilon}}.\bar{E} = \bar{0} \quad (10)$$

Where $\bar{\bar{\varepsilon}}$ is a dielectric tensor of plasma such as:

$$\bar{\bar{\varepsilon}} = \bar{\bar{I}} + \frac{i\bar{\bar{\sigma}}}{\varepsilon_0 \omega} \quad (11)$$

In general the wave is propagates perpendicularly to the exterior magnetic field $\left(\vec{k} \perp \bar{B}_0\right)$ and one gets in this case the system of equations:

$$\begin{bmatrix} \varepsilon_1 & -i\varepsilon_2 & 0 \\ i\varepsilon_2 & \varepsilon_1 - N^2 & 0 \\ 0 & 0 & \varepsilon_3 - N^2 \end{bmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0 \quad (12)$$

The refractive index [1] is defined by:

$$N = \frac{kc}{\omega} \quad (13)$$

And the elements of relative dielectric permittivity $\bar{\bar{\varepsilon}}$ are giving by the following expression:

$$\varepsilon_1 = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} \quad (14)$$

$$\varepsilon_2 = -\frac{\omega_{ce}}{\omega}\frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} \quad (15)$$

$$\varepsilon_3 = 1 - \frac{\omega_{pe}^2}{\omega^2} \quad (16)$$

With:

- The electronic plasma pulsation [2] is given by $\omega_{pe} = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$ and it is schematized on the Fig. 1.

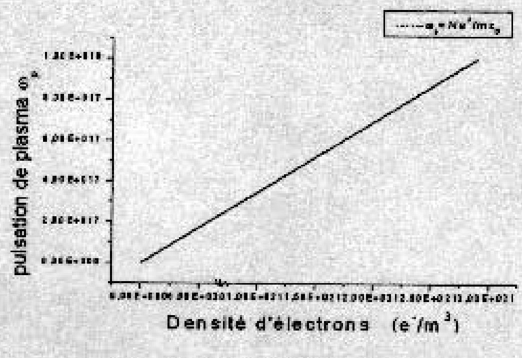

Fig. 1. The electronic plasma pulsation

The electronic cyclotron pulsation [4] is given by $\omega_{ce} = \frac{eB}{m_e}$ and it is schematized on the Fig. 2.

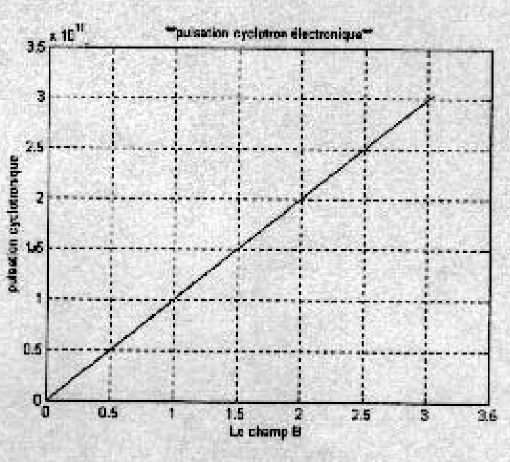

Fig. 2. The electronic cyclotron pulsation

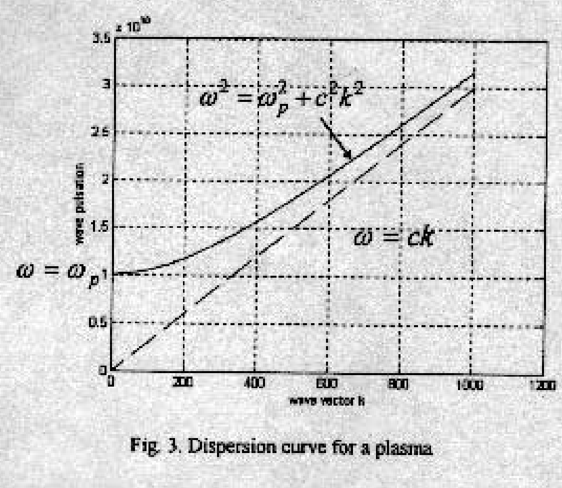

Fig. 3. Dispersion curve for a plasma

In Fig. 3 it is shown that for $\omega > \omega_{pe}$, we have propagation and for $\omega < \omega_{pe}$, we have an evanescent wave.

- $\vec{k} // \vec{E} \Rightarrow \omega^2 = \omega_p^2$ Plasma oscillation.
- $\vec{k} \perp \vec{E} \Rightarrow \omega^2 = k^2c^2 + \omega_p^2$ Electromagnetic wave.

The system of equation (12) presents no trivial solutions if its determinant is null. One gets so the general scattering relation:

$$\left(\varepsilon_3 - N^2\right)\left(\varepsilon_1 - N^2\varepsilon_1 - \varepsilon_2^2\right) = 0 \quad (17)$$

This relation can be developed in N power under the shape:

$$AN^4 - BN^2 + C = 0 \quad (18)$$

With A, B and C are the constants. This equation admits two solutions corresponding to two modes of polarization: ordinary mode and extraordinary mode.

III. The Mode of the Polarization

III.1. Ordinary Mode

The ordinary wave is a transverse wave, of straight polarization. Its electric field is parallel to the exterior magnetic field B_0 [5], like in the Fig. 4.

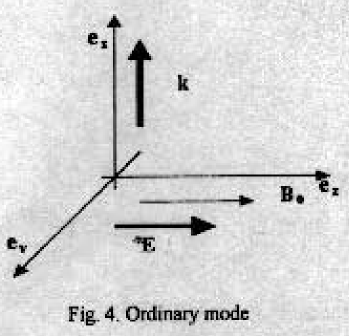

Fig. 4. Ordinary mode

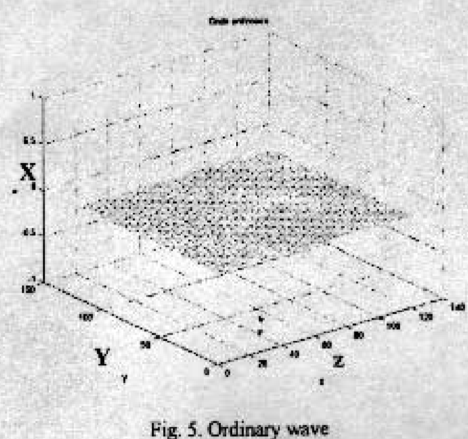

Fig. 5. Ordinary wave

In this case, the solution of equation (17) is $\varepsilon_3 = N^2$, therefore:

$$N_O^2 = 1 - \frac{\omega_{pe}^2}{\omega^2} \quad (19)$$

The electronic plasma pulsation ω_{pe} is not depends of the electronic density of plasma. So, in ordinary mode, the propagation of the wave depends only of the density of plasma.

III.2. Extraordinary Mode

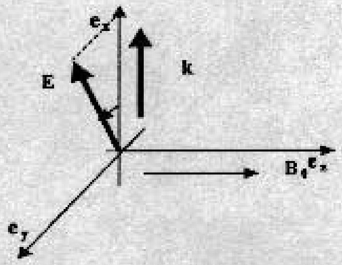

Fig. 6. Extraordinary mode

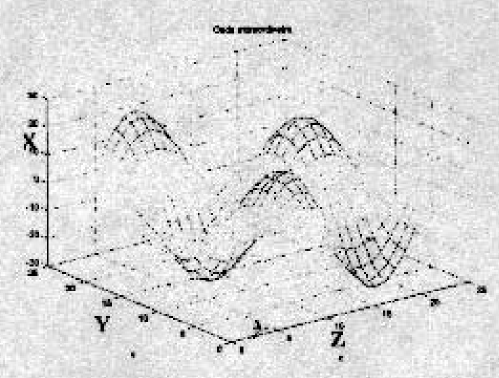

Fig. 7. Extraordinary wave

In Fig. 7 the extraordinary wave is polarized elliptically and its electric field is perpendicular to B_0 [5].

This mode is defined by $\varepsilon_1^2 - N^2\varepsilon_1 - \varepsilon_2^2 = 0$, where:

$$N_x^2 = 1 - \frac{\omega_{pe}^2}{\omega^2}\frac{\left(\omega^2 - \omega_{pe}^2\right)}{\left(\omega^2 - \omega_{pe}^2 - \omega_{ce}^2\right)} \quad (20)$$

The index of refraction is depending of electronic plasma pulsation and electronic cyclotron pulsation that depend at its tower of the exterior magnetic field. Therefore in mode-X [6] the propagation of the wave depends on the density and the magnetic field.

IV. Cut-Off and Resonance

For the N solutions of the equation (18), one gets the characteristic of the wave plasma. So one finds a cut-off or a resonance [7].

IV.1. Cut-Off

The frequency of cut-off is the frequency for which the index of refraction equal zero $N = 0$ corresponds to the amplitude of electric field $|E| \approx 0$ .This last separate a domain of frequency where the wave is propagates $(k^2 > 0)$ and a domain where the wave is evanescent $(k^2 < 0)$. In the case of the mode-O, the frequency of cut-off corresponds to the electronic frequency of plasma:

$$F_c = F_{pe} \tag{21}$$

The wave is propagate so much that its frequency is superior to the electrônic frequency of plasma and when frequency plasma becomes superior, the refractive index is imaginary pure, the wave becomes evanescent.

IV.2. Cut-Off

The resonance is defined by an index of refraction that offers toward the infinite N→ ∞, what corresponds to a number of waves infinite k and a null speed of phase of wave. It creates a transfer of energy between the wave and the middle and the wave is absorbed.

The ordinary mode doesn't possess a resonance, contrarily to extraordinary mode [6].

V. Transfer of Energy

Transfers of energy are generally described with the help of the instantaneous Poynting theorem. While using equations of Maxwell [3], one gets:

$$\vec{\nabla}.\left(\vec{E}\Lambda\vec{H}\right)+\left[\vec{B}.\frac{\partial\vec{H}}{\partial t}+\vec{E}.\frac{\partial\vec{D}}{\partial t}\right]=-\vec{j}_{ext}.\vec{E} \tag{22}$$

In a solid, it is easy to separate the physical significance of these terms:

The first term of the left member represents the electromagnetic energy variation.

The second is provoked by currents of polarization and magnetization, coming from the displacement of the atom, of cores around their position of equilibrium en presence of the applied field. There is a reversible energy transfer.

The right term present the current of conduction, and the transfer of energy is irreversible. On the other hand, generally, one can suppose that the free course means is very small in front of the length of wave, what means that the equilibrium between fields and the middle is almost-immediate.

Contrarily in plasma, the situation is more complicated; there is not a clear separation between the current of conduction and the current of polarization. All the charge is free, but contributes to the polarization. It implies that there is not equilibrium between fields and the middle on only one period of the wave and it will be necessary therefore to work on the means quantities on a certain number of periods. An energtic analysis to the ladder of the period doesn't have a physical sense.

One uses an eikonal shape for all oscillating quantities. Thus, one writes the electric field as:

$$\vec{E}=\frac{1}{2}\left[\vec{E}_0 e^{i\psi}+\vec{E}_0^{*} e^{-i\psi}\right] \tag{23}$$

Where $\vec{E}_0^{*}$ is the conjugate of $\vec{E}_0$. An eikonal function can be approximated by:

$$\psi = \vec{k}.\vec{r} - \omega.t \tag{24}$$

If one considers the multiplication of two quantities $\vec{A}$ and $\vec{B}$ of which the shape data by (23), one finds:

$$\left\langle \vec{A}.\vec{B}\right\rangle = \frac{1}{2}\Re\left(\vec{A}_0 + \vec{B}_0^{*}\right) \tag{25}$$

Applying the mean formulates of a product to the (25), one find:

$$\left\langle\vec{\nabla}.\left(\vec{E}\Lambda\vec{H}\right)\right\rangle+\left[\left\langle\vec{B}.\frac{\partial\vec{H}}{\partial t}\right\rangle+\left\langle\vec{E}.\frac{\partial\vec{D}}{\partial t}\right\rangle\right]=-\left\langle\vec{j}_{ext}.\vec{E}\right\rangle \tag{26}$$

One gets, then the theorem of Poynting:

$$\frac{\partial W}{\partial t}+\vec{\nabla}.\left(\vec{S}_0+\vec{T}\right)=-P_{abs} \tag{27}$$

Where:

- The electromagnetic energy density W is given by

$$W \equiv \frac{1}{2}\left[\frac{B_0^2}{\mu_0}+\vec{E}_0^{*}\frac{\partial}{\partial\omega}\left(\omega\overset{=h}{\varepsilon}\right).\vec{E}_0\right] \tag{28}$$

- The kinetic flux T is :

$$T = -\frac{\omega}{2}\vec{E}_0^{*}.\frac{\partial\overset{=h}{\varepsilon}}{\partial k}E_0 \tag{29}$$

The vector of Poynting $\vec{S}_0$ has the following relation:

$$\vec{S}_0 = \frac{1}{\mu_0}\Re\left(\vec{E}_0^*\Lambda\vec{B}_0\right) \quad (30)$$

- And the absorbed power is given by :

$$P_{abs} = \omega\vec{E}_0^*\overline{\overline{\varepsilon}}_a.\vec{E}_0 \quad (31)$$

The dielectric tensor is:

$$\overline{\overline{\varepsilon}} = \overline{\overline{\varepsilon}}^h + i\overline{\overline{\varepsilon}}^a \quad (32)$$

Where $\overline{\overline{\varepsilon}}^h$ the hermiteen part and $\overline{\overline{\varepsilon}}^a$ the anti-hermiteen.

VI. Conclusion

In this article, we solved the problem of propagation of a monochromatic plane electromagnetic wave in plasma perpendicularly to the exterior magnetic field.

By solving the relation of scattering for fixed ω, one can extricate a lot of information on characteristics of the wave in the considered middle. While looking for solutions for N one can define a cut-off for a null index of refraction or a resonance when the index offers toward the infinite.

In the case where the index is complex, there are a propagated modes, ordinary and extraordinary if $N^2 > 0$ and one obtained evanescent mode if $N^2 < 0$.

A transfer of energy settles between the wave and plasma is characterized by the theorem of Poynting. This theorem translated the relation between the electromagnetic energy density, the vector of poynting, the kinetic flux presenting the energy of the wave carried by the movement of particles and the strength dissipated by the wave.

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