The Magneto-Optic Rotation in Magnetised Plasma Study of Magneto-Optic Isolator

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ABSTRACT
In this article, we propose a study of the propagation of an electromagnetic field in a magnetized plasma that puts in game the study of a magneto-optic phenomenon of magnetic rotational polarization results of the interaction of the electromagnetic field with the considered medium that is not other that magneto-optic rotation or the rotation of Faraday [1]. This effect is used extensively to solve different problems notably by the techniques laser. Among these most important applications, the magneto-optic isolator. The problem is solved according to a main method of resolution founded on the calculation of the eigenvalues and the eigenvectors, while using the model of the electron elastically bound from Maxwell equations.
Keywords: electromagnetic field, rotation, magneto-optic, isolator, elastically bound electron, Maxwell equations.

1. INTRODUCTION

Our study concerning the propagation of a wave hovers an electromagnetic field in a non linear magnetised plasma has for objective analyzing and the understanding of phenomena determining types of propagation observed in presence of nonlinear and anisotropic magnetic field; as well as the study of the optic properties of a plasma as dielectric material in presence of a static magnetic field.

2. POSITION OF PROBLEM

To realise this study, one considers an electromagnetic plane monochromatic wave, of the pulsation ω and wave vector $\vec{K} = K\vec{z}$, linearly polarized and propagating in a linear, homogeneous isotropic and no magnetic medium, organized of molecules possessing each two peripheral electrons, submitted to a static magnetic field parallel to the direction of propagation, and submitted to a field of force:

$$\vec{f} = -m\omega_0 \vec{s}\ , \tag{1}$$

where, $\vec{s}$ is the vector radius of the electron, ω_0 the proper pulsation and m masse of dispersive electron.

To solve this problem, one uses the model of the electron elastically bound (Lorentz model) [2], that consists that every dispersive electron is submitted to an elastic force applied by the core, therefore, the medium is submitted to a field of force of the same shape that the relation (1).

3. RESOLUTION OF PROBLEM

The electric and magnetic fields of the plane wave are:

$$\begin{cases} \vec{E} = \vec{E}_0 \exp j\left(\omega t - \vec{k}\,\vec{r}\right) \\ \vec{B} = \vec{B}_0 \exp j\left(\omega t - \vec{k}\,\vec{r}\right) \end{cases} \tag{2}$$

This model described the movement of the electron by a differential equation of the shape

$$m\left(\omega_0{}^2 - \omega^2\right)\vec{S} + je\omega B_0 \vec{S}\Lambda\vec{Z} = -e\vec{E} \tag{3}$$

The solution of this equation presents the displacement of this dispersive electron

$$\vec{s} = -\frac{e}{m}\frac{\vec{E}\left(\omega_0{}^2 - \omega^2\right) + j\omega\Omega\vec{Z}\Lambda\vec{E}}{\left(\omega_0{}^2 - \omega^2\right)^2 - \omega^2\Omega^2} \tag{4}$$

which drags a dipolar moment and therefore a polarization of the medium, where $\Omega = \frac{eB_0}{m}$ and

$$\vec{P} = \varepsilon_0 \frac{Ne^2}{\varepsilon_0 m}\frac{\vec{E}\left(\omega_0{}^2 - \omega^2\right) + j\omega\Omega\vec{Z}\Lambda\vec{E}}{\left(\omega_0{}^2 - \omega^2\right)^2 - \omega^2\Omega^2} \tag{5}$$

Vectors of polarization and electric displacement [3] can be written as follows:

$$\begin{cases} \vec{P} = \varepsilon_0 [\chi] \vec{E} \\ \vec{D} = \varepsilon_0 [\varepsilon_r] \vec{E} \end{cases}. \tag{6}$$

Therefore, one finds

$$[\varepsilon_r] = \begin{bmatrix} \varepsilon_1 & -j\varepsilon_2 & 0 \\ j\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_1 \end{bmatrix}, \tag{7}$$

with

$$\begin{cases} \varepsilon_1 = 1 + \dfrac{Ne^2}{\varepsilon_0 m} \dfrac{(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 - \omega^2 \Omega^2} \\ \varepsilon_2 = \dfrac{Ne^2}{\varepsilon_0 m} \dfrac{\Omega \omega}{(\omega_0^2 - \omega^2)^2 - \omega^2 \Omega^2} \end{cases}. \tag{8}$$

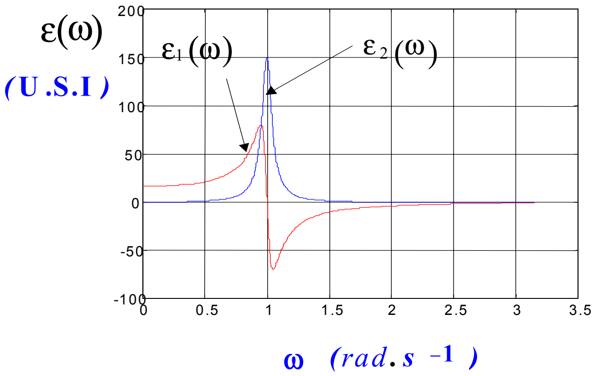


Figure 1. ε_1 and ε_2 in function of wave pulsation.

One defines the relation of scattering

$$k^2 \vec{E} = \frac{\omega^2}{c^2} [\varepsilon_r] \vec{E} \tag{9}$$

which lead to the resolution of an equation of the eigenvalues

$$n^2 \vec{E} = [\varepsilon_r] \vec{E}. \tag{10}$$

With $k = n\dfrac{\omega}{c}$ and n^2 being the corresponding eigenvalue of the restriction $[\varepsilon_r]$. The equation (10) of the eigenvalues provides the following relation of scattering :

$$n^2 = \varepsilon_1 \pm \varepsilon_2 . \tag{11}$$

This relation implies that the medium possesses two index of refraction [4]. Therefore the medium is birefringent and the eigenvectors are states of two circular wave polarization left $(n^2 = \varepsilon_1 - \varepsilon_2)$ and right $(n^2 = \varepsilon_1 + \varepsilon_2)$. The relation (11) is written also by replacing ε_1 and ε_2 by their values

$$k2 = \frac{\omega^2}{c^2} \left[1 + \frac{Ne^2}{\varepsilon_0 m} \frac{1}{\omega_0^2 - \omega^2 \pm \omega \Omega} \right] \tag{12}$$

with $\omega < \omega_0$ and $|\omega \Omega| < \omega_0 - \omega^2$, k is real, then the medium is transparent.

4. ANALYSIS AND UNDERSTANDING OF THE PHENOMENON

A plane monochromatic wave polarized linearly following the axis (ox) and propagating in the sense positive of (oz) is transmitted in the medium in $z = 0$, it takes the shape:

$$\vec{E} = E\,\vec{x} = E_0 \cos \omega t\ \vec{x}\ . \tag{13}$$

It is the electric field of the incidental wave polarized linearly of amplitude E_0.

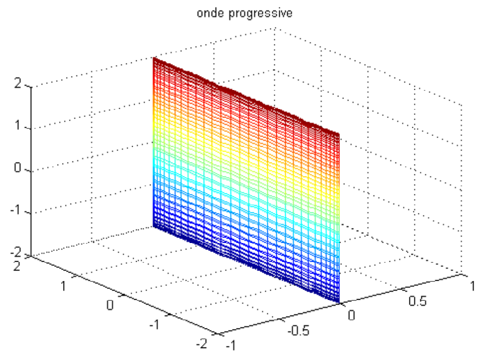


Figure 2. Wave of field $\vec{E} = E_0 \cos \omega t\ \vec{x}$.

For $z \neq 0$, this wave transmitted in the medium decomposes in two circular inverse waves of the same amplitudes $\frac{E_0}{2}$ and with different speeds. It causes a dephasing between these waves, due to the magnetic field that they are affected differently [5].

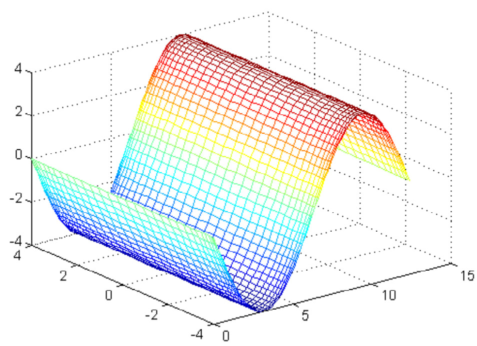


Figure 3. Wave polarized circular left E_+.

On the figure 3, the wave which is polarized circularly left represents the field:

$$E_+ = \frac{E_0}{2}\left[\cos(\omega t - k'z)\vec{x} + \sin(\omega t - k'z)\,\vec{y}\right] \tag{14}$$

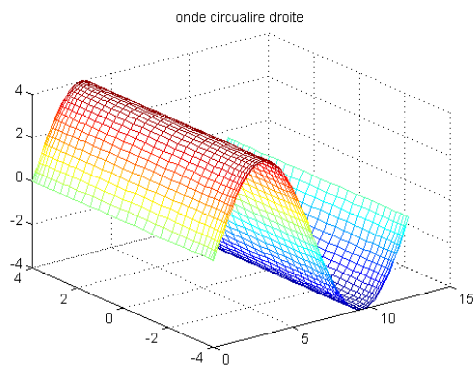


Figure 4. Wave polarized circular right E+ .

On the figure 4, the wave which is polarized circularly right represents the field :

$$E_- = \frac{E_0}{2}\left[\cos(\omega t - k''z)\vec{x} - \sin(\omega t - k''z)\,\vec{y}\right] \tag{15}$$

The summation of these two circular waves gives polarized linearly wave (Fig. 5):

$$\vec{E}(z,t) = E_+ + E_- , \tag{16}$$

$$\vec{E}(z,t) = \left[E_0 \cos(\omega t - \frac{k'+k''}{2} z) \cos \frac{k''-k'}{2} z \right] \vec{x} + \left[E_0 \cos(\omega t - \frac{k'+k''}{2} z) \sin \frac{k''-k'}{2} z \right] \vec{y} . \tag{17}$$

For a given constant direction z , the wave results by the superposition of two inverse circular waves polarized linearly, but in another plan making an angle with the one of the incidental initial wave.

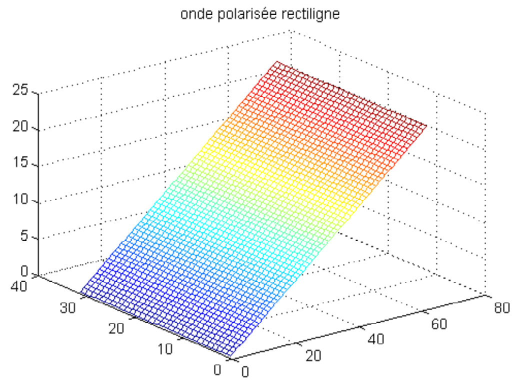


Figure 5. Superposition of two circular waves inverse.

4.1 Determination of the rotation angle

For a rotation of angle ϕ around of (oz). One calculus of the dephasing between the two states of polarization:

$$\phi = \frac{\omega}{c} \frac{n(-) - n(+)}{2} z , \tag{18}$$

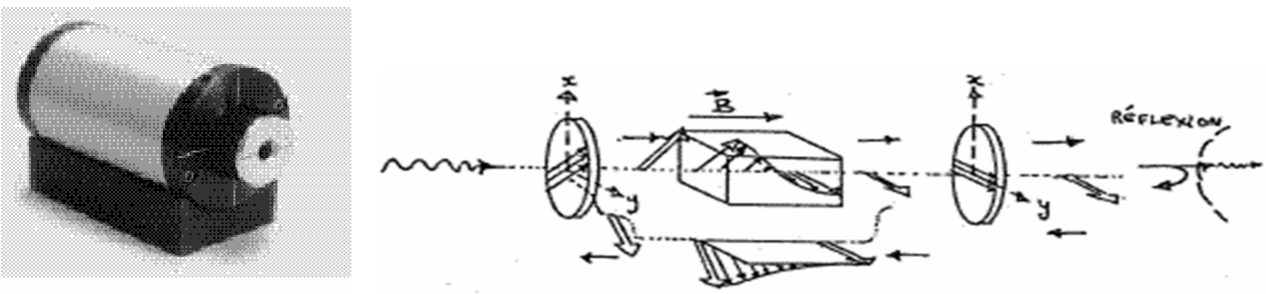
$$\phi = \frac{\omega}{c} \frac{1}{2n_0} \frac{Ne^2}{\varepsilon_0 m} \frac{\omega\Omega}{(\omega_0^2 - \omega^2)^2} z . \tag{19}$$

It is the effect of the magneto-optic rotation that disappears in the absence of the magnetic field.

5. MAGNOTO-OPTIC ISOLATOR

During last years, the magneto-optic rotation is used extensively to solve different problems, notably in the field of the laser techniques and telecommunications system. Among its important applications, we find the spatial modulator of light, optical circulator, current transformer etc.… and the magneto-optic isolator that is the object of this study as device used in telecommunication systems [6].

An isolator is a device as show the figure 6*a* that does not allow light to propagate in an unique direction. The principle of the isolator is that the angle of polarization rotation is the same the even that light propagating parallel or inversely parallel to the sense of the magnetic field.



(a) One of forms of magneto-optic isolator.

(b) Diagram of magneto-optic isolator

Figure 6. Magneto-optic isolator.

The diagram show that the polarizer fix the angle of polarization of the incidental light to the value $-\theta$. The magnetic field is adjusted in such a way that the angle of polarization of the electric field turns of θ. This reflexive wave recovers the polarization with a global rotation 2θ , it is sufficient to chosen $\theta = \pi/4$ so that this wave crossing with the polarizer is not transmitted. The polarizer lets only pass a given straight polarization and the magnetic field in a material encapsulate in a cylinder is produced by a coil, no represented on this figure (Fig. 6*b*).

5.1 Operation of an isolator

We can distinguish two modes : the forward mode and the reverse mode [7]

The forward mode (Fig. 7)
Laser light, whether or not polarized, enters the Input Polarizer and becomes linearly polarized, say in the vertical plane (0°). It then enters the Faraday rotator rod, designed to rotate the plane of polarization (POP) by 45°. It hen exits through the Output Polarizer whose axis is at 45°. The light leaves the Isolator, and reflections occur. This reflected light constitutes feedback.

The reverse mode (Fig. 8)
This feedback re-enters the Isolator, back through the Output Polarizer where it is repolarized at 45°. It then passes back through the rotator rod and is further rotated by another 45°, making a total of 90° with respect to the input polarizer (0°). It is seen that the light is extinguished here. Thus, we have succeeded in isolating the laser from its own reflections.

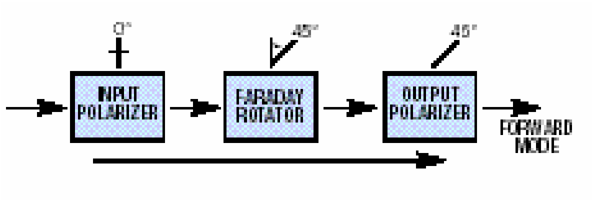


Figure 7. The forward mode.

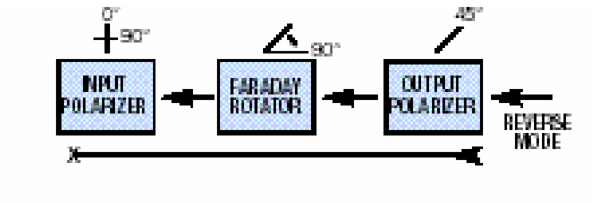


Figure 8 . The reverse mode

5. CONCLUSIONS

In this article, we solved the problem of propagation of an electromagnetic plane monochromatic wave, in a linear, homogeneous isotropic plasma, in presence of a static magnetic field according to a main method of resolution founded on the research of the eigenvalues and the eigenvectors while using the model of the electron elastically bound. This conducted to the study of a magnetic rotational polarization phenomenon, the effect to turn the plan of polarization of the wave of a certain angle. The application of the static magnetic field in the medium made appear several important optic properties:

- The nonlinear response of the material described in term of dielectric permittivity and electric susceptibility that are non linear complex tensors.
- The variation of the structure of the material (helical symmetry of the magnetic field), as well as the anisotropy and the birefringence by the magnetic field.
- The important property of the amplification of the Faraday effect that marks the difference between the natural rotational power and the forced rotational power.
- This last property is used in laser applications permits the synthesis and the realization of materials of high Verdet constant of $V = \phi / lB_0$ [2] and the manufacturing of magneto-optic devices as the isolator.

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