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Study of A Nonlinear Electronic Circuit Supplied With A Photovoltaic Generator

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Abstract

In this work we use a photovoltaic generator to supply a nonlinear electronic circuit working under an external control. The circuit allows a transfer of the power delivered by a photovoltaic generator, which is a finite source of energy, towards a receiver which is, in our case, a direct current machine.

The nonlinear differential system characterizing the circuit is linearized using the optimal derivation. A study, of the solutions behavior and the external control, is presented and discussed.

Keywords: Photovoltaic Generator – Nonlinear System – Optimal Linear System – Optimal Derivative – External Force

Introduction

Except the nuclear energies and geothermal, the sun is at the origin of the near total sources of energies used by humanity for its food, domestic and industrial needs: biomass, wind, hydraulics, fossil fuels. The sun thus provides each day to the Earth, by its radiation, the equivalent of several thousands of times the total power consumption of humanity for its activities of today. [1-4]

The contribution of solar energy would thus be enough largely to feed terrestrial consumption; the only problem is to convert solar energy into a usable energy.

The photovoltaic effect which consists in transforming directly heat into electricity without a mechanical engine makes is a solution to this problem [5-10]. This direct transformation of heat uses associated photovoltaic cells generally in parallel to provide the desired voltage and current. Consequently, the photovoltaic generators are produced by association of a large number of unit cells.

In this work, we use a photovoltaic generator to supply a nonlinear electronic circuit working under an external force. This electronic circuit allows the transfer of power from a photovoltaic generator, a receiver which is in our case a direct current (dc) machine. The nonlinear differential equation characterizing the circuit is described by a system of nonlinear ordinary differential equations of the form

$$\begin{cases} \dfrac{dx}{dt} = F(x(t), u(t)) \\ x(0) = x_0 \end{cases} \tag{1}$$

where

$x = (x_1, \cdots, x_n)$ is the unknown function,

$u = (u_1, \cdots, u_n)$ is the external excitation which can be constant (continuous) or function of time (periodic),

$F = (f_1, \cdots, f_n)$ is a given function on an open subset Ω of IR^n,

One applies the method of optimal derivation, introduced by Arino –Benouaz [11, 14, 15, 17, and 19], in order to linearize this nonlinear system, and one makes a comparison starting from the quadratic error between the nonlinear system and the optimal linear obtained application [19].

Position of the Problem

We consider the system (1) of nonlinear ordinary differential equations with the following assumptions:

1) $F(0,0) = 0$.

2) F is γ Lipchitz continuous,

3) the spectrum $\sigma(DF(x))$ is contained in the set $\{z : \mathrm{Re}\ z < 0\}$ for every $x \neq 0$, in a neighborhood of 0, for which $DF(x)$ exists.

Our purpose can be formulated in order to find a linear ordinary differential equation given as:

$$\begin{cases} \dfrac{dx}{dt} = \tilde{A}\, x + \tilde{B}\, u \\ x(0) = x_0 \end{cases} \tag{2}$$

Approaches the non linear equation (1) under the same initial conditions and so that the functional

$$G(A, B) = \int_0^{+\infty} \| F(x(t), u(t)) - A\, x(t) - B\, u(t) \|^2\, dt \tag{3}$$

is minimal

where $(\tilde{A}, \tilde{B}) \in M_n(IR^n)$ is to be determined.

The considered problem is an optimization in the least square sense. We aim to replace the initial non linear equation with a linear one; i.e.: search a solution approximation of the system (1) using the solution of the system (2). The minimization of the functional $G(A,B)$, with respect to A and B, is done to the respect to the solutions stemmed from the initial point and lead to the regime solution when $t \to +\infty$.

Formalism

Differentiating $G(A,B)$ with respect to A along a function x, and with respect to B along a function u yields

$$\begin{cases} dG(A)\alpha = 2\int_0^{+\infty} \left\langle \begin{matrix} Ax(t)+Bu(t)- \\ F(x(t),(u(t))), \alpha\, x(t) \end{matrix} \right\rangle dt \\ dG(B)\beta = 2\int_0^{+\infty} \left\langle \begin{matrix} Ax(t)+Bu(t)- \\ F(x(t),(u(t))), \beta\, u(t) \end{matrix} \right\rangle dt \end{cases} \tag{4}$$

for every matrixes α and β, in particular for the matrixes such that

$$\begin{cases} \alpha_{l,m} = 1\ ;\ \alpha_{ij} = 0 \\ \beta_{l,m} = 1\ ;\ \beta_{ij} = 0 \end{cases} \text{ if } (i,j) \neq (l,m) \tag{5}$$

Letting

$$\begin{cases} \Gamma_A(x) = \int_0^{+\infty} [x(t)][x(t)]^T\, dt \\ \Gamma_B(u) = \int_0^{+\infty} [u(t)][u(t)]^T\, dt \\ \phi_A(u,x) = \int_0^{+\infty} [u(t)][x(t)]^T\, dt \\ \phi_B(x,u) = \int_0^{+\infty} [x(t)][u(t)]^T\, dt \\ \Psi_A(x,u) = \int_0^{+\infty} [F(x(t),u(t))][x(t)]^T\, dt \\ \Psi_B(x,u) = \int_0^{+\infty} [F(x(t),u(t))][u(t)]^T\, dt \end{cases} \tag{6}$$

After some algebraic manipulations [16, 18], we obtain

$$\begin{cases} A\,\Gamma_A(x) + B\,\Phi_A(u,x) = \Psi_A(x,u) \\ A\,\Phi_B(u,x) + B\,\Gamma_B(u) = \Psi_B(x,u) \end{cases} \tag{7}$$

This will allow us to write the matrixes A and B as

$$\begin{cases} A = [\Psi_A(x,u) - (B)\phi_A(u,x)][\Gamma_A(x)]^{-1} \\ B = [\Psi_B(x,u) - (A)\phi_B(x,u)][\Gamma_B(u)]^{-1} \end{cases} \tag{8}$$

We have implicitly assumed that matrixes Γ_A and Γ_B are non singular and consequently A and B are uniquely defined if $\Gamma_A(x)$ and $\Gamma_B(x)$ can be inverted.

Calculus Procedure

The computation presented above will iteratively be used. We assume that the successive matrixes A_j and B_j are stable and their spectrum lies in $\{z : \operatorname{Re} z < 0\}$. The initial matrixes A_0 and B_0 are the Jacobian matrixes of F at x_0 such that DF(x) exists, and at u_0 such that DF(u) exists; x_0 and u_0 are the initial values of x and u, respectively.
Considering the system given by (1), the computational procedure can be summarized as follows:

First step

Computing $\begin{cases} A_0 = DF(x_0) \\ B_0 = DF(u_0) \end{cases}$

Second step

Computing A_1 and B_1 from the solution of equation

$$\begin{cases} \dfrac{dx}{dt} = A_0\, y(t) + B_0\, v(t) \\ y(0) = x_0 \end{cases} \tag{9}$$

which is

$$y(t) = e^{A_0 t} x_0 + \int_0^t e^{(t-s)A_0} B_0\, v(s)\,ds \tag{10}$$

by minimizing the functional

$$G(A,B) = \int_0^{+\infty} \left\| F(y(t), v(t)) - A\, y(t) - B\, v(t) \right\|^2 dt \tag{11}$$

A_1 and B_1 are uniquely determined by the system (8), where x is replaced by y and v(t) is the excitation at time t.

Apart the initial matrixes, the matrixes determined by the procedure are not Jacobian matrixes of F at a given point. It is necessary that the above conditions of this study should be satisfied at each step. Let us assume that this is true so, the procedure works as follows:

<u>Third step</u>

Assuming that $A_1,\cdots,A_{j-1}$ and $B_1,\cdots,B_{j-1}$ have been computed, to compute A_j from A_{j-1} and B_j from B_{j-1}; first we solve the following system

$$\begin{cases} \dfrac{dx}{dt} = [A_{j-1}]\, y(t) + [B_{j-1}]\, v(t) \\ y(0) = x_0 \end{cases} \tag{12}$$

The solution y_j of this equation is

$$y_j(t) = e^{A_{(j-1)}t} x_0 + \int_0^t e^{(t-s)A_{(j-1)}} B_{(j-1)}\, v_j(s) ds \tag{13}$$

The minimization of the functional

$$G_j(A,B) = \int_0^{+\infty} \left\| F(y_j(t), v_j(t)) - A\, y_j(t) - B\, v_j(t) \right\|^2 dt \quad (14)$$

gives

$$\begin{cases} [A_j] = \begin{bmatrix} \int_0^{+\infty} [F(y_j(t), v_j(t))][y_j(t)]^T dt - \\ (B_j)(\phi(v_j, y_j)) \end{bmatrix} [\Gamma(y_j)]^{-1} \\ [B_j] = \begin{bmatrix} \int_0^{+\infty} [F(y_j(t), v_j(t))][v_j(t)]^T dt - \\ (A_j)(\phi(y_j, v_j)) \end{bmatrix} [\Gamma(v_j)]^{-1} \end{cases} \tag{15}$$

If the sequences (A_j, B_j) converge, then the limit $(\tilde{A}, \tilde{B})$ is, by definition, the optimal approximation of $F(x(t), u(t))$ at the point of (x_0, u)

Application

In this section, we present an application related the procedure of the optimal derivative of a system governed by nonlinear ordinary differential equations with external excitation. This enables us to test the obtained results by quadratic error analysis.

We consider the case of a nonlinear electronic circuit comprised of three state variables and working with an external force as shown in the following circuit.

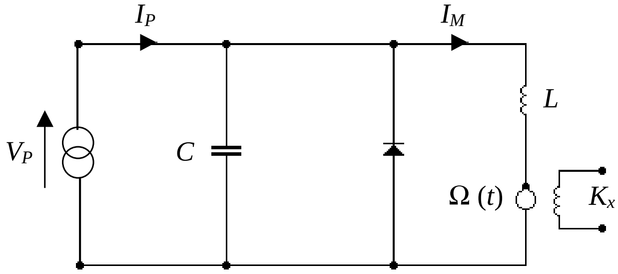

Figure 1: A standard electrical non linear circuit comprised of a photovoltaic generator and a DC motor.

This electronic circuit allows the transfer of electrical power from the photovoltaic generator, towards a receiver which is in this case a D.C machine.

The origin of the non linearity is provided by the characteristic function of the photovoltaic generator:

$$I_P = I_O - I_S \left[\exp\left(\frac{q\,V_P}{k\,T} \right) - 1 \right] \tag{16}$$

where:
I_O is the photocurrent proportional to the illumination.
I_P is the current corresponding to voltage V_p.
I_S is the saturation current
To be more general, let

- The generator voltage $V_P = x$
- The current of the DC motor $I_M = y$
- The rotation speed of the motor $\Omega = z$

The circuit parameters, used in our numerical application, are:

$$\begin{aligned} &C = 500 \cdot 10^{-6}\ \mathrm{F} \quad && K_x = 0{,}5 \\ &J = 0{,}001\ \mathrm{kg\,m^2} \quad && K_r = 0{,}1 \\ &L = 0{,}1\ \mathrm{H} && \end{aligned} \tag{17}$$

we put $\gamma = \frac{q}{k\,T}$ in the characteristic (16) of the photovoltaic generator and use the following numerical values of the constants :
$\gamma = 0{,}54\,V^{-1}$, $I_O = 2A$, $I_S = 1{,}28 \cdot 10^{-5}$ A ,
Hence

$$I_P = 2 - 1{,}28 \cdot 10^{-5} \left[\exp\left(0{,}54\,V_P\right) - 1 \right] \tag{18}$$

The application of the Kirschoff’s laws leads to the following equations:

$$\frac{d\,V_P}{d\,t} = \frac{I_P}{C} - \frac{I_M}{C} \tag{19}$$

$$\frac{d\,I_M}{d\,t} = \frac{V_P}{L} - \frac{R_m}{L} I_M - \frac{K_x}{L}\Omega \tag{20}$$

Where R_m is the resistance of the armature of the machine, K_x is a proportionality factor of the electromotive force of the machine with D.C. current.
The moment theorem enables us to write the following equation:

$$\frac{d\,\Omega}{d\,t} = \frac{K_x}{J} I_M - \frac{K_r}{J}\Omega \tag{21}$$

where:

J is the moment of inertia of the machine
K_r is the proportionality factor of the resistive couple.
If we put $I_P = f(V_P)$ and x, y, z in equations (19), (20) and (21), we obtain the following set of equations:

$$\begin{cases} \frac{dx}{dt} = u(t) - \frac{I_S}{C} e^{\gamma x} - \frac{y}{C} \\ \frac{dy}{dt} = \frac{x}{L} - \frac{R_m}{L} y - \frac{K_x}{L} z \\ \frac{dz}{dt} = \frac{K_x}{J} y - \frac{K_r}{J} z \end{cases} \tag{22}$$

where u(t) represent the excitation :

$$u(t) = \left[\frac{I_O + I_S}{C}\right] [H(t)] = [4000,0256]\ [H(t)] \tag{23}$$

H(t) is the Heaviside function.

Then the system (22) is written, by taking account of the values of the circuit components, in the form:

$$\begin{cases} \frac{dx}{dt} = 4000,0256\, H(t) - 0.0256\ e^{0,54\ x} - 2000\, y \\ \frac{dy}{dt} = 10\ x - 120,454\, y - 5\ z \\ \frac{dz}{dt} = 500\ y - 100\ (21)\ z \end{cases} \tag{24}$$

This system of nonlinear ordinary differential equations is solved referring to the initial conditions:

$$(x_0, y_0, z_0) = (22.1465\ ,\ 0,\ 0) \tag{25}$$

Corresponding to the studied circuit of the nonlinear characteristic of the photovoltaic generator.

By applying the procedure developed before, and after 7 iterations ($\varepsilon = 10^{-6}$), the optimal linear system is written as:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -194,01 & 201,50 & 23,05 \\ 10 & -120,454 & -5 \\ 0 & 500 & -100 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} u(t) \tag{26}$$

We will compare the results obtained and those given by the linearization in Taylor series to those of the nonlinear system. The classical linearization gives:

$$\dot{x} = A\,x + B\,u \tag{27}$$

with : $A = \left.\frac{\partial F}{\partial x}\right|_{\substack{x = x_0 \\ u}}$ and $B = \left.\frac{\partial F}{\partial u}\right|_{\substack{x = x_0 \\ u}}$

Figures (2), (3) and (4) represent respectively the solutions $(x(t), y(t), z(t))$ as a function of time for the solution of the systems (24), (26) and (27).

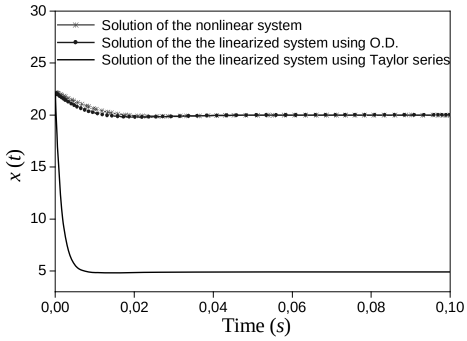

Figure 2: Variation of the solution x (t) as a function of time for the initial conditions (x_0, y_0, z_0).

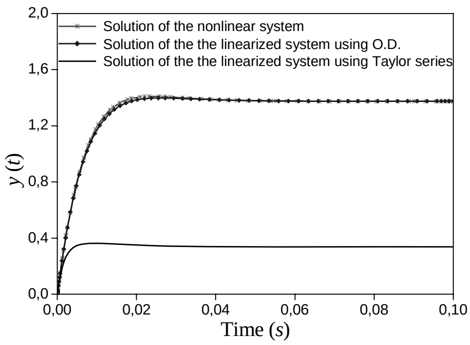

Figure 3: Variation of the solution y (t) as a function of time for the initial conditions (x_0, y_0, z_0).

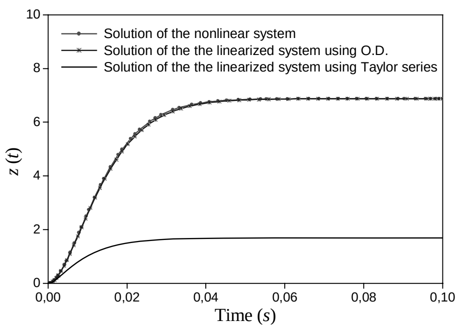

Figure 4: Variation of the solution z(t) as a function of time for the initial conditions (x_0, y_0, z_0).

Comparison

In this part, we compare the results obtained by a quadratic error analysis. This analysis is done by using the following relationship

$$\mathrm{Er} = \sum_{n=1}^{n} \left\| x(t) - \tilde{y}(t) \right\|^2 \tag{28}$$

x (t) represents the solution of the nonlinear system,

$\tilde{y}(t)$ represents the solution of the linear system obtained starting from the optimal derivative.

Figure (5) represents the quadratic error as a function of time.

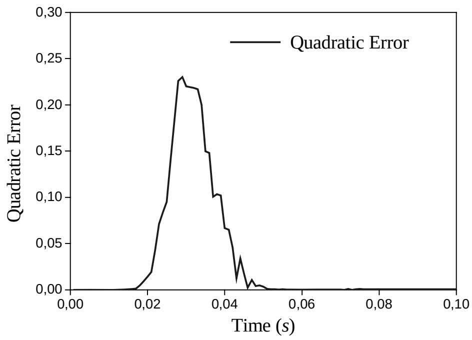

Figure 5: The quadratic error as a function of time between the nonlinear system (24) and the optimal linear system (26).

It is noticed that the solution given by the optimal derivative is of the same order of magnitude as that given by the nonlinear equation. It brings the system after excitation, towards its point of operation. The curves given by the expansion in Taylor series are far from the exact solution. Hence the procedure of optimal derivation is better. This is demonstrated by the quadratic error which reaches its maximum at time t = 0,029 s; thus 23% remains small.

This error almost vanishes l when $t \geq 0{,}05 s$ i.e. when the solutions reach the operation point of the electronic circuit. At this level the solution of the nonlinear system is superimposed with that of the optimal linear system.

Conclusion

The behaviour of a nonlinear electronic circuit supplied with a photovoltaic generator is presented and discussed. Our study shows clearly that the approximation obtained by the optimal derivation gives satisfactory results compared to the exact results while respecting the dynamics of the initial problem. It is noticed, also, that the solutions obtained converge when $t \to +\infty$. This work developed a good demonstration of the photovoltaic solar energy conversion, provided an overview of photovoltaic generator operation and analysed the photovoltaic behaviour as a power generation technology.

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