The Magneto-Optical Properties of a Dielectric In The Presence Of Static Magnetic Field

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Abstract: In this article, we propose studying the magneto-optical properties result to the presence of static magnetic field in dielectric. At this reason we consider an electromagnetic wave enters the considered medium, its magneto-optical properties may change. Namely, its intensity (amplitude), polarization, velocity, wavelength, etc. may alter.

Keys word — Magneto-optical properties, magnetic field, electromagnetic waves.

1. Introduction

The decrease in velocity of electromagnetic wave in matter is caused by the fact that all materials have a refraction index, the ratio of the velocities of electromagnetic wave measured in vacuum and in the given material.

To realize this work concerning the study of the magneto-optical properties of dielectric, we considered the propagation of a plane electromagnetic wave in an insulating nonlinear medium, which has for objective the analysis and comprehension of the phenomena determining the types of propagation observed in the presence of non-linearity; as well as the study of the magneto-optical properties of a dielectric material in the presence of a static magnetic field.

For that, we study the action of such medium on the propagation of an electromagnetic wave which penetrates in this one.

2. Model of Lorenz

We considers an electromagnetic plan monochromatic wave, with pulsation ω and vector of wave $\vec{K} = K\vec{z}$, linearly polarized and propagating in a linear, homogeneous isotropic and non magnetic middle [1], constituted of molecules possessing each two peripheral electrons, submitted to a static magnetic field parallel to the direction of propagation $\vec{oz}$, and submitted to the field of force of the shape

$$\vec{f} = -m\omega_0 \vec{s} \qquad (1)$$

Where, $\vec{s}$ is the vector radius of the electron, ω_0 the proper pulsation, m the mass of spread dispersive electron.

To solve this problem, we use the model of the electron elastically bound [2], that consists that every electron dispersive is submitted to strength elastic applied by the core.

For an insulating, linear, homogeneous isotropic and non magnetic middle, the system of equation of Maxwell that describes the electromagnetic field

$$\begin{cases} \overrightarrow{Div}\,\vec{D}=0 & \overrightarrow{Rot}\,\vec{E}=-\dfrac{\partial \vec{B}}{\partial t} \\ \overrightarrow{Div}\,\vec{B}=0 & \overrightarrow{Rot}\,\vec{B}=-\dfrac{\partial \vec{D}}{\partial t} \end{cases} \qquad (2)$$

The electric and magnetic fields of the plane wave are

$$\begin{cases} \vec{E} = \vec{E}_0 \exp j\left(\omega t - \vec{k}\,\vec{r}\right) \\ \vec{B} = \vec{B}_0 \exp j\left(\omega t - \vec{k}\,\vec{r}\right) \end{cases} \qquad (3)$$

This model described the movement of the electron by a differential equation of the shape

$$m\left(\omega_0{}^2 - \omega^2\right)\vec{S} + je\omega B_0 \vec{S}\Lambda\vec{Z} = -e\overrightarrow{E} \qquad (4)$$

The solution of this equation presents the displacement of this dispersive electron

$$\vec{s} = -\frac{e}{m}\frac{\vec{E}\left(\omega_0{}^2 - \omega^2\right) + j\omega\Omega\vec{Z}\Lambda\overrightarrow{E}}{\left(\omega_0{}^2 - \omega^2\right)^2 - \omega^2\Omega^2} \qquad (5)$$

Who products a dipolar moment and therefore a polarization of the middle, where $\Omega = \dfrac{eB_0}{m}$.

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$$\vec{P} = \varepsilon_0 \frac{Ne^2}{\varepsilon_0 m} \frac{\vec{E_1}\left(\omega_0^2 - \omega^2\right) + j\omega\Omega \vec{Z}\Lambda\vec{E}}{\left(\omega_0^2 - \omega^2\right)^2 - \omega^2\Omega^2} \quad (6)$$

3. Tensors of electric permittivity and electric susceptibility:

Vectors of polarization and electric displacement [3] can be written under the following shape

$$\begin{cases} \vec{P} = \varepsilon_0 [\chi]\vec{E} \\ \vec{D} = \varepsilon_0 [\varepsilon_r]\vec{E} \end{cases} \quad (7)$$

Then, we find

$$[\varepsilon_r] = \begin{bmatrix} \varepsilon_1 & -j\varepsilon_2 & 0 \\ j\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_1 \end{bmatrix} \quad (8)$$
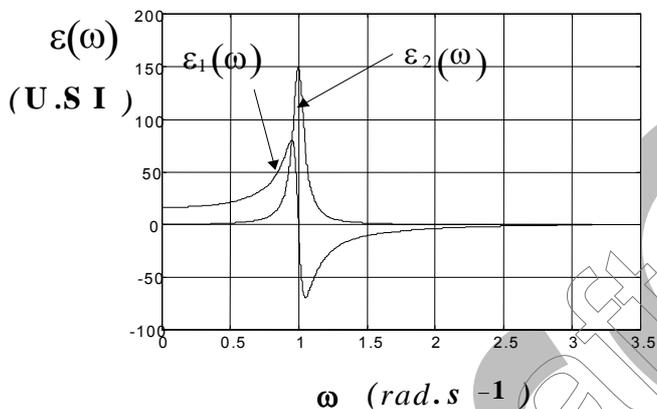

Figure1: ε_1 and ε_2 in function of pulsation wave.

With :

$$\begin{vmatrix} \varepsilon_1 = 1 + \dfrac{Ne^2}{\varepsilon_0 m} \cdot \dfrac{\left(\omega_0^2 - \omega^2\right)}{\left(\omega_0^2 - \omega^2\right)^2 - \omega^2\Omega^2} \\ \\ \varepsilon_2 = \dfrac{Ne^2}{\varepsilon_0 m} \dfrac{\Omega\omega}{\left(\omega_0^2 - \omega^2\right)^2 - \omega^2\Omega^2} \end{vmatrix} \quad (9)$$

3. Relation of scattering and magnetic birefringence

With Maxwell's equations in notation complex [4]

$$\begin{cases} \overrightarrow{RotE} = j\omega\vec{B} \\ \overrightarrow{RotB} = \dfrac{k}{\omega}\vec{Z}\Lambda\vec{E} \end{cases} \quad (10)$$

We defines the relation of scattering

$$k^2\vec{E} = \frac{\omega^2}{c^2}[\varepsilon_r]\vec{E} \quad (11)$$

This conducted to the resolution of an equation

$$n^2\vec{E} = [\varepsilon_r]\vec{E} \quad (12)$$

Where $k = n\dfrac{\omega}{c}$ and if a vector of the plan (xoy) of components $(x,y,0)$; this relation shows that $\vec{E}$ it is eigenvector of the restriction $[\varepsilon_r]$, n^2 being the eigenvalue correspondent [2]. From the equation (11), one deduces that $y = \pm jx$. This middle has a special property: it refraction index is different for left and right circularly polarized wave. This phenomenon is called circular birefringence [5].

The equation of the eigenvalues provides a relation of scattering of the shape

$$n^2 = \varepsilon_1 \pm \varepsilon_2 \quad (13)$$

This relation implies that the middle possesses two indications of refraction (double indices) [6] therefore the middle is birefringent and the eigenvectors are the polarization states of two circular wave left $\left(n^2 = \varepsilon_1 - \varepsilon_2\right)$ and right $\left(n^2 = \varepsilon_1 + \varepsilon_2\right)$.
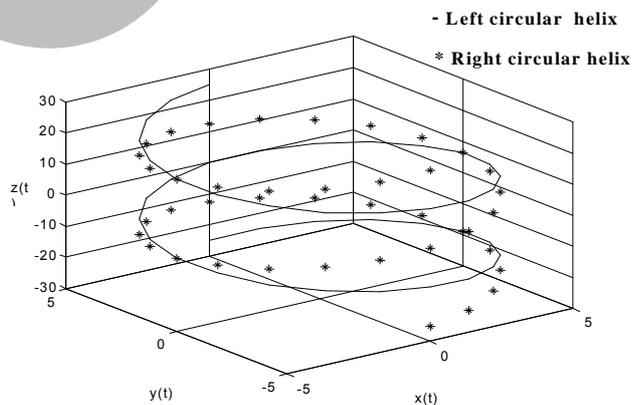

Figure 2: Propagation of field according to a left or right circular helix.

During the propagation of the plane wave according to (oz), the extremity of the field $\vec{E}$ described either the left circular helix or the right one according to the corresponding own value. It is schematized well on the Fig. 2.

The relation (13) can written also, with replacing

ε_1 and ε_2 by their values, we find

$$k^2 = \frac{\omega^2}{c^2}\left[1 + \frac{Ne2}{\varepsilon 0 m}\frac{1}{\omega_0^2 - \omega^2 \pm \omega\Omega}\right] \qquad (14)$$

With $\omega\langle\omega_0$ and $|\omega\Omega|\langle\omega_0^2 - \omega^2$, k is real, then the

middle is transparent.

4. Rotational power and optical activity stimuled

A wave plane monochromatic polarized linearly following the axis (ox) and propagating in the sense positive of (oz) is transmitted in the middle in z=0, it takes the shape:

$$\vec{E} = E\vec{x} = E0\cos\omega t\,\vec{x} \qquad (15)$$

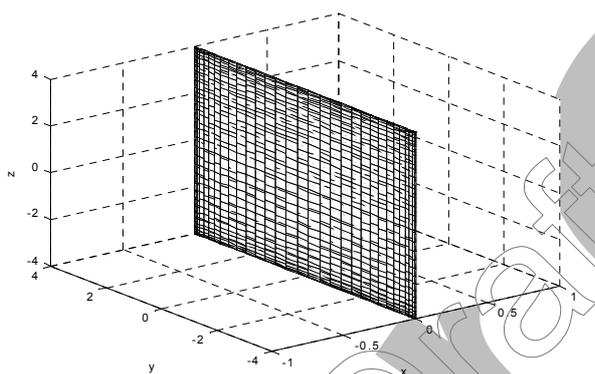It is the electric field of the incidental wave polarized linearly of amplitude $E0$.

Figure 3. The wave of the field $\vec{E} = E_0 \cos\omega t\,\vec{x}$.

For z given, this wave transmitted in the middle divides in two circular waves reverse equal-amplitudes $\frac{E_0}{2}$; they propagate with the different velocities, what causes a difference phase between these waves. Grace to the magnetic field that they are affected differently [7].

The wave polarized circular left represents the field

$$E_+ = \frac{E_0}{2}\left[\cos(\omega t - k'z)\vec{x} + \sin(\omega t - k'z)\,\vec{y}\right] \qquad (16)$$

The right circular polarized wave represents the field

$$E_- = \frac{E_0}{2}\left[\cos(\omega t - k''z)\vec{x} - \sin(\omega t - k''z)\,\vec{y}\right] \qquad (17)$$

The superposition or the summation of the two circularly polarized components is a plane-polarized wave with a plane of polarization makes an angle with the one of the initial wave as Fig. 4 shows it.

$$\vec{E}(z,t) = E_+ + E_- \qquad (18)$$

$$\vec{E}(z,t) = \frac{E_0}{2}\left[\cos(\omega t - k'z)\vec{x} + \sin(\omega t - k'z)\,\vec{y}\right] + \frac{E_0}{2}\left[\cos(\omega t - k''z)\vec{x} - \sin(\omega t - k''z)\,\vec{y}\right] \qquad (19)$$

$$\vec{E}(z,t) = \begin{bmatrix} E_0\cos(\omega t - \frac{k'+k''}{2}z)\cos\frac{k''-k'}{2}z \\ E_0\cos(\omega t - \frac{k'+k''}{2}z)\sin\frac{k''-k'}{2}z \\ 0 \end{bmatrix} \qquad (20)$$

For Z given, $\vec{E}$ has a constant direction, the wave result from the superposition of two waves opposite circular is polarised linearly. It is the phenomenon of rotator polarization magnetic disappears in absence of the magnetic field $\vec{B}0$.
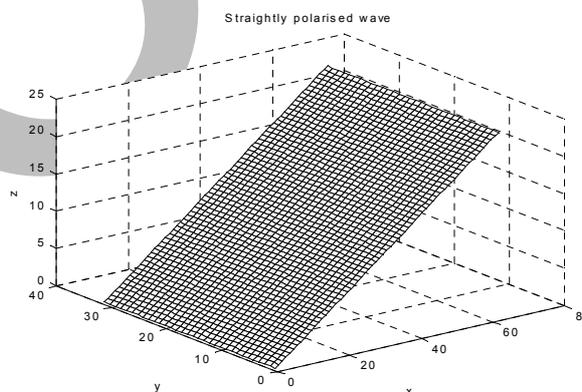

Straightly polarised wave

Figure 4. Superposition of two waves polarized circular inverse.

5. Determination of the rotation angle

The direction of the electric field in z deducts of the one of z=0 for a rotation of angle φ around (oz).
We calculus the difference of phase between the two states of polarization [8].

$$\varphi = \frac{\omega}{c}\frac{n(-) - n(+)}{2}z \qquad (21)$$

$$\varphi = \frac{\omega}{c}\frac{1}{2n_0}\frac{N\,e^2}{\varepsilon_0 m}\frac{\omega\Omega}{(\omega_0^2 - \omega^2)^2}z \qquad (22)$$

It is the phenomenon of magnetic rotary polarization that disappears in absence of the magnetic field.

According to Faraday $\varphi = VlB_0$ [4]. This is implies that the constant of Verdet takes the value

$$V = \frac{\omega}{c} \frac{1}{2n_0} \frac{N e^2}{\varepsilon_0 m^2} \frac{\omega}{(\omega_0^2 - \omega^2)^2} \qquad (23)$$

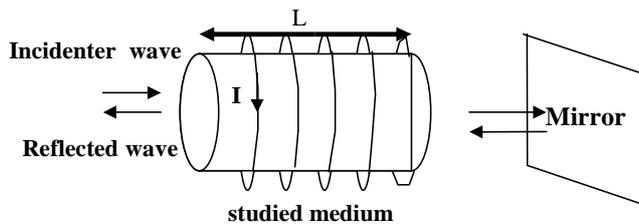## 6. The amplification of Faraday Effect

Figure 5. Amplification of the Faraday effect

We get this amplification by the reflection of the wave considered on the mirror as the shows the Fig. 5.

During a round-trip, the angle of rotation is doubled, what demonstrates a very important property of this effect that is used in applications lasers, "the amplification of the Faraday effect by multiple reflections". The total rotation of the direction of the electric field is therefore 2φ during a round-trip [9].

5. Conclusion

In this communication, we solved the problem of propagation of a plane, monochromatic electromagnetic wave in such medium in the presence of a static magnetic field according to a principal method of resolution founded on research of the eigenvalues and the eigenvectors, by using the model of the electron elastically dependent, and then we obtain the following result:

-The nonlinear response of material describes in term of permittivity dielectric and electric susceptibility which are nonlinear complex tensors.

-Variation in the structure of the material (helicoids symmetry of the magnetic field), as well as the anisotropy and birefringence caused by the magnetic field.

- Any linearly polarized light wave can be obtained as a superposition of a left circularly polarized and a right circularly polarized wave, whose amplitude is identical.

-Circular birefringence rotates the plane of polarization of plane-polarized electromagnetic wave.

-The important property of the amplification of the Faraday Effect which marks the difference between the natural optical activity and the caused one. This property is very used in the laser applications allows the synthesis and the realization of materials of great constant of Verdet.

This phenomenon is governed by the law of Verdet which explained like result of circular birefringence according to the kinematics' theory of Fresnel.

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